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CP Violation in the B-Meson System

Abstract

This review of CP violation in the B-meson system appeared as an appendix to the Princeton U. high-energy-physics FY97 grant renewal proposal. It is based in part on preprint Princeton/HEP/92-09 (Sept. 20, 1992), which contains many references.

1 The Four Classes of CP Violation of Neutral B Mesons

CP violation can be very prominent in the B-meson system because the relevant CP-violating phases in the CKM matrix occur in first order. This is in contrast to the K-meson system where CP-violating phases arise in the relevant matrix elements only in higher order.

In decays of neutral B-mesons to self-conjugate final states there are four classes of CP violation, as can be motivated by the Wolfenstein representation of the CKM matrix. We note the location of the matrix elements that have imaginary parts (to first order):

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \mathrm{Re} & \mathrm{Re} & \mathrm{Im} \\ \mathrm{Re} & \mathrm{Re} & \mathrm{Re} \\ \mathrm{Im} & \mathrm{Re} & \mathrm{Re} \end{pmatrix}.$$

The phase of V_{td} enters in B_d (but not B_s) mixing due to top-quark exchange in the box diagram.

The phase of V_{ub} enters in $b \to u$ (but not $b \to c$) decays.

Hence there are 4 classes of CP violation in decays of neutral B's, as listed in Table 1.

Table 1: The four classes of CP violation in the neutral B-meson system.

Class	Parent	Quark Transition	Example	CP-Violating Phase
1	B_d	$b \to c$	$B_d \to J/\psi K_S$	$\varphi_1 = \varphi(V_{td})$
2	B_d	$b \to u$	$B_d \to \pi^+\pi^-$	$\varphi_2 = \varphi(V_{td}) + \varphi(V_{ub})$
3	B_s	$b \to u$	$B_s \to \rho K_S$	$\varphi_3 = \varphi(V_{ub})$
4	B_s	$b \to c$	$B_s \to J/\psi\phi$	$\varphi_4 = 0$

For decays of neutral B's to CP eigenstates f, the observable effect is the decay asymmetry

$$A(t) = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})} = \sin 2\varphi_i \sin xt,$$

where φ_i is the relevant phase of the CKM matrix element listed above, $x = \Delta M/\Gamma$ is the mixing parameter, and t is the proper time of the decay measured in units of the lifetime.

As there are three classes of nonzero asymmetries, we can make three measurements of the two CKM phases, and hence overconstrain the Standard Model.

This insight is also commonly expressed via the unitarity triangle. But it is important to note that the Standard Model predicts a null effect in a fourth class of decays, which are perhaps more accessible at hadron colliders than at e^+e^- colliders.

The above argument is reviewed in greater detail in the subsections after next.

2 The Superweak Model

The superweak model is often used as a vehicle for discussions of alternatives to the Standard Model of CP violation. In this model CP-violating effects are due to a new interaction that manifests itself only in the mixing of a neutral meson and its antiparticle. The effect is small, but different in principle for each type of neutral meson. In the superweak model there are only two classes of CP violation in the neutral B mesons, one for B_d, and another for B_s. This contrasts with the four classes discussed above in the Standard Model. Thus the observation that either

1. $\sin 2\varphi_1 \neq \sin 2\varphi_2$, or that
2. $\sin 2\varphi_3 \neq \sin 2\varphi_4$

would contradict the superweak model. In the Standard Model it is possible that $\sin 2\varphi_1 = \sin 2\varphi_2$, but it is extremely unlikely that both equalities would hold simultaneously.

A third confrontation between the superweak and Standard Models is possible with B mesons:

3. The Standard Model suggests that there will be small but nonzero CP-violating asymmetries in the decay rates of B^+ and B^- mesons, while the superweak model predicts a null effect.

3 The Need for Interference in CP-Violating Processes

In the Standard Model, CP violation in a process described by a single graph manifests itself only as a phase factor. If the amplitude for a single graph $B \to f$ is written

$$A(B \to f) \equiv A_f = |A_f|\, e^{i\phi_W} e^{i\delta_S}, \tag{1}$$

where ϕ_W is a phase due to the weak interaction, and δ_S is a phase due to strong final-state interactions, then the CP conjugate process has amplitude

$$A(\overline{B} \to \overline{f}) \equiv \overline{A}_{\overline{f}} = |A_f|\, e^{-i\phi_W} e^{i\delta_S}. \tag{2}$$

Hence CP violation cannot be discerned as a rate difference between a decay and its CP-conjugate decay if only a single graph contributes to the amplitude.

CP violation can only be revealed in *total-rate measurements* of $B \to f$ and $\overline{B} \to \overline{f}$ when there is interference between two or more decay amplitudes with differing weak phases *and* differing strong phases. To verify the last remark, consider the case where two graphs contribute to a decay, written as

$$A(B \to f) = |A_1|\, e^{i\phi_1} e^{i\delta_1} + |A_2|\, e^{i\phi_2} e^{i\delta_2}, \tag{3}$$

so the CP-conjugate decay has amplitude

$$A(\overline{B} \to \overline{f}) = |A_1|\, e^{-i\phi_1} e^{i\delta_1} + |A_2|\, e^{-i\phi_2} e^{i\delta_2}. \tag{4}$$

The corresponding decay rates are given by

$$\Gamma(B \to f) = |A_1|^2 + |A_2|^2 + 2\,|A_1|\,|A_2| \cos(\phi + \delta), \tag{5}$$

and

$$\Gamma(\overline{B} \to \overline{f}) = |A_1|^2 + |A_2|^2 + 2\,|A_1|\,|A_2| \cos(\phi - \delta), \tag{6}$$

where $\phi = \phi_1 - \phi_2$ and $\delta = \delta_1 - \delta_2$. Only if both ϕ and δ are nonvanishing can the interference term be determined from measurements of the two decay rates.

Even if this condition is satisfied the strong-interaction phase difference δ and the magnitudes $|A_f|$ and $\left|\overline{A}_{\overline{f}}\right|$ will not typically be known, and the CP-violating phase cannot be determined.

4 Neutral B-Meson Decays to CP Eigenstates

The most well-known method for extracting CP-violating phases uses neutral B mesons that decay to CP eigenstates f. In this case

$$|\overline{f}\rangle \equiv CP|f\rangle = \eta|f\rangle \qquad \text{where} \qquad \eta = \begin{cases} +1 & CP(\text{even}) \\ -1 & CP(\text{odd}) \end{cases}. \tag{7}$$

The decay amplitude can be written

$$A(B^0 \to f) = |A|\, e^{-i\phi_D} e^{i\delta}, \tag{8}$$

where δ is a strong-interaction phase, and the weak-interaction phase ϕ_D depends on whether the decay proceeds via a $\overline{b} \to \overline{c}$ or $\overline{u}$ transition:

$$\phi_D = \begin{cases} \phi_{cb} = 0, & b \to c \\ \phi_{ub}, & b \to u \end{cases}. \tag{9}$$

Due to mixing, a particle that was created as a B^0 (or $\overline{B}^0$) at $t = 0$ has evolved by time t to the state we label as $B^0(t)$ (or $\overline{B}^0(t)$) according to

$$\begin{aligned} B^0(t) &= e^{-iMt}e^{-t/2}[\cos(xt/2)|B^0\rangle + ie^{2i\phi_M}\sin(xt/2)|\overline{B}^0\rangle], \\ \overline{B}^0(t) &= e^{-iMt}e^{-t/2}[ie^{-2i\phi_M}\sin(xt/2)|B^0\rangle + \cos(xt/2)|\overline{B}^0\rangle], \end{aligned} \tag{10}$$

where we measure time in units of the relevant B lifetime, $x = \Delta M/\Gamma$ is the mixing parameter, and the relative amount of $|B^0\rangle$ and $|\overline{B}^0\rangle$ in the weak eigenstate B_S^0 is given by a pure phase coming from the mixing box diagram where

$$\phi_M = \begin{cases} \phi_{td}, & \text{for } B_d^0 \\ \phi_{ts} \approx 0, & \text{for } B_s^0 \end{cases} \tag{11}$$

Following eq. (2) we can write the amplitude for the CP-conjugate process as

$$A(\overline{B}^0 \to \overline{f}) = \eta A(\overline{B}^0 \to f) = |A|\, e^{i\phi_D}e^{i\delta}, \quad \text{and hence} \quad A(\overline{B}^0 \to f) = \eta\, |A|\, e^{i\phi_D}e^{i\delta}, \tag{12}$$

using eq. (7). Combining eqs. (8-12) with (10) we arrive at the time-dependent decay rates

$$\begin{aligned} \Gamma(B^0(t) \to f) &\propto |A|^2\, e^{-t}[1 - \eta\sin(xt)\sin 2(\phi_M + \phi_D)], \\ \Gamma(\overline{B}^0(t) \to f) &\propto |A|^2\, e^{-t}[1 + \eta\sin(xt)\sin 2(\phi_M + \phi_D)]. \end{aligned} \tag{13}$$

If, as we have assumed, only a single graph contributes to $B^0 \to f$, then there is only a single strong-interaction phase δ in both this and the conjugate reaction $\overline{B}^0 \to f$. This single phase does not appear at all in the interference term in eq. (13).

Both ϕ_M and ϕ_D can take on two values depending on the decay considered, according to eqs. (9) and (11), so there are four classes of phase angles can be explored by measurements of neutral B decays to CP eigenstates, as listed in Table 2. Classes 1, 2 and 3 provide measurements of φ_1, φ_2 and φ_3, respectively, of the unitarity test. Class-4 decays should show very little CP violation, but not necessarily zero, as they depend on V_{ts} which has a CP-violating phase at higher order (see eq. (15)). Any difference in the size of the CP violation between class 1 and class 2, or between class 3 and class 4 would indicate that the superweak model is not the source of that effect.

The class-1 decay $B_d^0 \to J/\psi K_S^0$ is particularly easy to trigger on and identify, and may provide the first evidence for CP violation in the B system. The most prominent class-2 and -3 decays, $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to \rho^0 K_S^0$, respectively, both have smaller branching ratios and in particular it may prove elusive to measure φ_3 with $B_s^0 \to \rho^0 K_S^0$.

Another potential difficulty is that with the exception of $B_d^0 \to J/\psi K_S^0$, all other decays to CP eigenstates have admixtures of penguin diagrams with different weak phases than the dominant tree diagram. Hence it is useful to have other procedures than the present method to measure φ_2 and φ_3.

Table 2: The 23 basic neutral-B decays to CP eigenstates. The graphs associated with each decay mode are shown in Fig. 1. The subscripts F, S, and D refer to CKM-favored (amplitude $\propto \lambda^2$), -suppressed ($\propto \lambda^3$), and -doubly-suppressed ($\propto \lambda^4$), respectively. The weak-interaction phase $\phi_M + \phi_D$ is shown in parentheses after each graph type, where ϕ_M is the phase due to mixing and ϕ_D is the phase due to $\overline{b}$-quark decay. Penguin graphs (V-VII) are included in classes 1-4 if they lead to the same final state as the nominal graphs for that class, even though their topology is different. Classes 1a and 4a are pure penguin graphs. Within each class the modes are ranked roughly in order of decreasing branching ratio. A final-state π^0 could be replaced by an η, ρ^0, ω, *etc.*, and a J/ψ could be replaced by an η_c, χ, ψ', *etc.*, but final states with two spin-1 particles must be analyzed according to method 6.

Class	B^0	$\overline{b} \to \overline{q}$	Modes	Graph($\phi_M + \phi_D$)
1	B_d^0	$\overline{b} \to \overline{c}$	$J/\psi K^0_{S,L}$	$\mathrm{II}_F(\phi_{td})$, $\mathrm{VI}_F(\phi_{td})$
			D^+D^-	$\mathrm{I}_S(\phi_{td})$, $\mathrm{IV}_S(\phi_{td})$, V_S, VII_S
			$J/\psi\pi^0$	$\mathrm{II}_S(\phi_{td})$, VI_S
			$D_s^+D_s^-$	$\mathrm{IV}_S(\phi_{td})$, V_S
			$\phi K^0_{S,L}$	$\mathrm{VI}_F(\phi_{td})$, $\mathrm{VII}_F(\phi_{td})$
2	B_d^0	$\overline{b} \to \overline{u}$	$\pi^+\pi^-$	$\mathrm{I}_S(\phi_{td}+\phi_{ub})$, $\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S, VII_S
			$\pi^0\pi^0$	$\mathrm{II}_S(\phi_{td}+\phi_{ub})$, $\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S, VI_S, VII_S
			$\rho^0 K^0_{S,L}$	$\mathrm{II}_D(\phi_{td}+\phi_{ub})$, $\mathrm{VI}_F(\phi_{td})$, $\mathrm{VII}_F(\phi_{td})$
			$D^0\overline{D}^0$	$\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S
			K^+K^-	$\mathrm{IV}_S(\phi_{td}+\phi_{ub})$, V_S
3	B_s^0	$\overline{b} \to \overline{u}$	$\rho^0 K^0_{S,L}$	$\mathrm{II}_S(\phi_{ub})$, $\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$
			K^+K^-	$\mathrm{I}_D(\phi_{ub})$, $\mathrm{IV}_D(\phi_{ub})$, V_F, VII_F
			$\phi\pi^0$	$\mathrm{II}_D(\phi_{ub})$, VI_F
			$\pi^+\pi^-$	$\mathrm{IV}_D S(\phi_{ub})$, V_F,
			$\pi^0\pi^0$	$\mathrm{IV}_D S(\phi_{ub})$, V_F,
4	B_s^0	$\overline{b} \to \overline{c}$	$D_s^+D_s^-$	I_F, IV_F, V_F, VII_F
			$J/\psi K^0_{S,L}$	II_S, $\mathrm{VI}_S(\phi_{td})$
			$D^0\overline{D}^0$	IV_F, $\mathrm{IV}_D(\phi_{ub})$, V_F, V_S
			D^+D^-	IV_F, V_F
			$K^0\overline{K}^0$	V_F, VII_F
1a	B_s^0	$\overline{b} \to \overline{s}$	$\phi K^0_{S,L}$	$\mathrm{VI}_S(\phi_{td})$, $\mathrm{VII}_S(\phi_{td})$
4a	B_d^0	$\overline{b} \to \overline{u}$	$\phi\pi^0$	VI_S
			$K^0\overline{K}^0$	V_S, VII_S

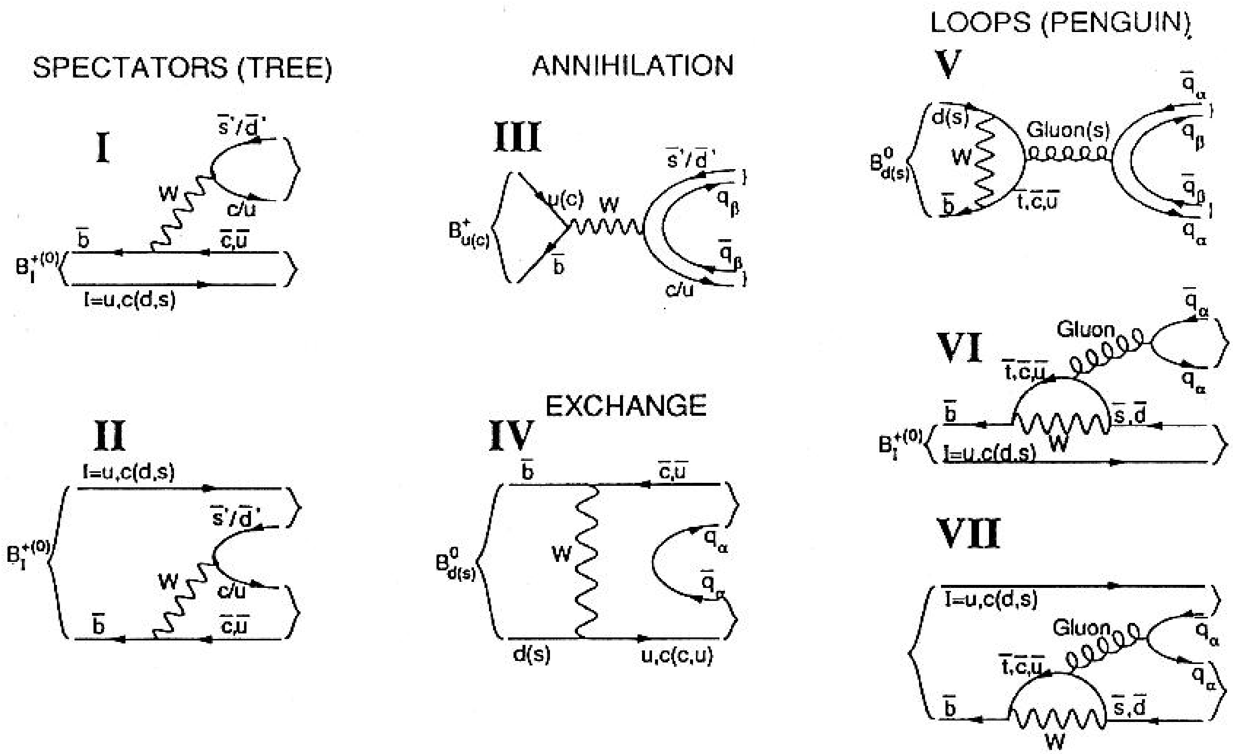

Figure 1: Seven graphs for the nonleptonic decays of B mesons.

5 Six Ways to Measure CP-Violating Phases in B Decays

The richness of approach to observing CP violation in B-meson decays is by no means restricted to the previous example. We briefly list six methods. In practice the largest signals will likely arise in the decays of neutral B-mesons to CP eigenstates.

1. **B decays to D^0X, $\overline{D}^0X$, and $D^0_{1,2}X$ where $X \neq \overline{X}$**

   When a B particle can decay both to D^0X and $\overline{D}^0X$ (and so $\overline{B}$ decays to both $\overline{D}^0\overline{X}$ and $D^0\overline{X}$), then the decays

   $$B \to D^0_{1,2}X, \qquad \text{and} \qquad \overline{B} \to D^0_{1,2}\overline{X}, \qquad \text{where} \qquad D^0_{1,2} \equiv \frac{D^0 \pm \overline{D}^0}{\sqrt{2}}, \tag{14}$$

   exhibit a CP-violating asymmetry. Measurement of the six (or eight) decay modes listed will permit isolation of the CP-violating amplitude, both in magnitude and phase.

   The final state D^0X need not be self conjugate, and it is actually desirable that it not be, so that no effects of mixing are present, and no tagging of the second B is needed.

Thus method 1 could be used at a symmetric e^+e^- collider without the penalty due to mixing of methods 2-6. This method works both for decays of B-mesons and b-baryons.

The general approach of methods 1-3 was largely anticipated by Carter and Sanda, but recent interest stems from the more specific formulation of Gronau and London. Method 1 as distinct from method 3 was first examined by Gronau and Wyler, with further discussions given by Dunietz. Application of method 1 to b-baryons was first discussed by Aleksan, Dunietz and Kayser.

If CP violation is found in such an analysis then it cannot be due to to superweak model, which postulates that CP violation occurs only in mixing of neutral mesons. Thus method 1 may be used to circumvent possible ambiguities in the use of method 4 to prove or disprove the superweak model.

2. **Neutral B-meson decays to f and $\overline{f}$ where $f \neq \overline{f}$**

   If a neutral B-meson decays to both a final state f and its CP-conjugate state $\overline{f}$, then the interference of amplitudes needed for measurable CP violation arises due to mixing (whether or not there is CP violation in the mixing). A time-dependent analysis of the four decay modes $B(\overline{B}) \to f, \overline{f}$ can isolate the CP-violating phase.

   Tagging of the particle-antiparticle character of the second B in the event is required.

   The original paper on method 2 is by Gronau and London. Discussion of method 2 as separate from method 3 was first been given by Aleksan *et al.* Method 2 is an improvement on earlier discussions by Du, Dunietz and Wu, and by Dunietz and Rosner in which only two of the four related decays were utilized.

3. **Neutral B-meson decays to D^0X, $\overline{D}^0X$, and $D^0_{1,2}X$ where $X = \overline{X}$**

   If a neutral B-mesons decays to both a final state D^0X and $\overline{D}^0X$ where X is self conjugate ($CP(X) \equiv \overline{X} = \pm X$), then methods 1 and 2 can be combined. In a case of interest two different CP-violating phases can be determined from the time-dependent analysis of six (or eight) related decay modes.

   As previously mentioned, method 3 was first discussed by Gronau and London.

4. **Neutral B-meson decays to CP eigenstates**

   If a neutral B-meson decays to a final state f that is a CP eigenstate, then as in method 2, CP violation becomes observable via the interference due to mixing. But since only a single final state is involved the strong-interaction phase does not appear. Thus we recover the well-known result that a time dependent analysis of the two modes $B(\overline{B}) \to f$ can isolate the CP-violating phases.

   The advantages of measuring decays to CP eigenstates were first noted by Bigi and Sanda. The important relation between decays to CP eigenstates and unitarity of the CKM matrix was first emphasized by Bjorken, and will be reviewed in the following subsection. The measurement of the three angles of the unitarity triangle by three specific decays to CP eigenstates was first proposed by Krawczyk *et al.*

5. B **decays to sets of final states related by isospin**

   In decays $B_u^+ \to f^+$ and $B_d^0 \to f^0$ where the final states each arise due to the inference of two amplitudes, and f^+ and f^0 are related by isospin, the CP-violating phase can be isolated by a detailed isospin analysis.

   The utility of the isospin analysis in removing uncertainties due to penguin diagrams in B decays was first demonstrated by Gronau and London. Further discussions have been given by Nir and Quinn, by Lipkin *et al.* and by Gronau.

6. **Angular analysis of** B **decays to mixtures of** CP **eigenstates**

   If a neutral B-meson decays to a self-conjugate state f, but this is not a pure CP eigenstate (as holds when f consists of two spin-1 mesons) method 4 cannot be carried out. However, a detailed analysis of the angular distribution of the secondary-decay products can separate the final state into CP(even) and CP(odd) components and the CP-violating phase extracted.

   Methods of angular analysis for B decays to mixtures of CP eigenstates have been presented for several years with recent discussion by Kayser *et al.*, by Dunietz *et al.*, and by Kramer and Palmer.

6 The Unitarity Test

In view of the variety of methods of measuring the phases of the CKM matrix elements it is useful to have an overall goal in pursuing an experimental program. This has been elegantly defined by Bjorken as a test of unitarity of the CKM matrix. This will provide a comprehensive test of the Standard Model view of CP violation as arising from phases in the transformation between the three generations of strong and weak quark base states.

We will discuss the CKM matrix in the Wolfenstein notation:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\lambda^2/2+\lambda^4/24 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2-\lambda^4(A^2/8-1/24) & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2+A\lambda^4(1/2-\rho-i\eta) & 1-A^2\lambda^4/2 \end{pmatrix}, \tag{15}$$

carrying the expansion in the parameter λ ($\approx$ the Cabibbo angle) to fourth order. From measurements of the B-meson lifetime it is known that $A \approx 1$. CP violation arises in the Standard Model because $\eta \neq 0$.

The unitarity of V_{CKM} implies that

$$\sum_k V_{ik}V_{jk}^* = \delta_{ij} = \sum_k V_{ki}V_{kj}^*. \tag{16}$$

Of these 18 conditions the one obtained using the first and third rows (or almost equivalently, the first and third columns) is especially suitable for testing via measurements of weak phase angles:

$$0 = V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} \approx V_{td} + \lambda V_{ts} + V_{ub}^*. \quad (17)$$

Regarding V_{td}, λV_{ts} and V_{ub} as vectors they form a closed triangle in the complex plane. On dividing their lengths by $A\lambda^3$, we obtain the picture of Figure 2 in the (ρ, η) plane.

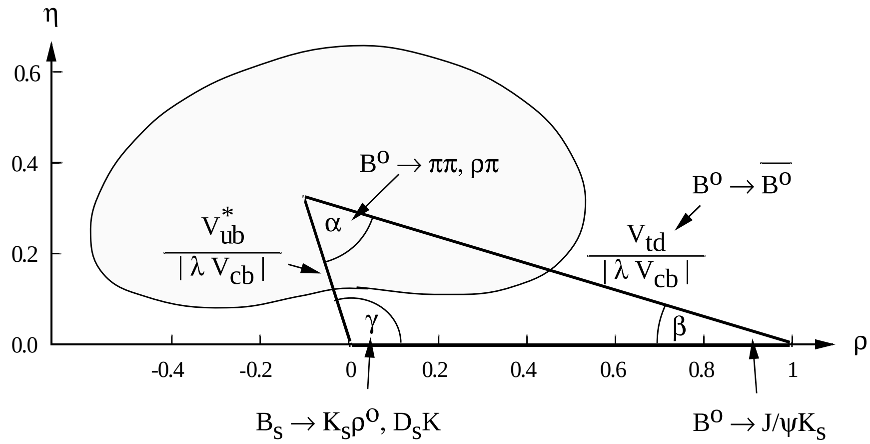

Figure 2: The unitarity triangle in the (ρ, η) plane. The phase angles φ_i used in the text relate to the angles in the figure according to $\varphi_1 = \beta$, $\varphi_2 = \alpha$ and $\varphi_3 = \gamma$. The shaded region indicates the present experimental uncertainty in the location of the vertex of the unitartity triangle.

The unitarity test then consists of measuring the magnitudes and phases of these three vectors to confirm that they form a closed triangle. It is anticipated that measurement of the magnitude of V_{td} via its role in the box diagram governing B_d^0 mixing will remain subject to theoretical uncertainties due to strong-interaction effects for some time to come. The insight of Bjorken was that a test of the closure of the unitarity triangle can be based on measurement of the three interior angles, φ_1, φ_2, φ_3, which should sum to π. The magnitudes of the three angles and the area $A^2\lambda^6\eta/2$ of the unitarity triangle are invariant under the choice of representation of the CKM matrix.

In the Wolfenstein parametrization the three angles φ_i can be related to phases of CKM matrix elements according to

$$\begin{aligned} \varphi_1 &= 2\pi - \phi(V_{td}) && \equiv 2\pi - \phi_{td}, \\ \varphi_2 &= \pi - \varphi_1 - \varphi_3 && = -\pi + \phi_{td} + \phi_{ub}, \\ \varphi_3 &= \phi(V_{ub}^*) = -\phi(V_{ub}) && \equiv -\phi_{ub}. \end{aligned} \quad (18)$$

A favorable theoretical result is that method 4, the study of neutral B decays to CP eigenstates, can in principle determine all three angles φ_i by measurement of three different decays. However, the anticipated difficulty in measuring ϕ_{ub} via decays such as $B_s^0 \to \rho^0 K_S^0$, due to the small branching ratios (and poor signal of B_s at e^+e^- colliders), has been a motivation to explore the additional methods of analysis of CP violation reviewed above.

7 The Einstein-Rosen-Podolsky Effect

If the B^0-$\overline{B}^0$ pair is produced in a C(odd) or C(even) combination, this quantum-mechanical correlation is maintained until both B's decay, even though they may be spatially separated, and they decay at different times. The complexity of such correlations was remarked by Einstein, Rosen, and Podolsky in a famous paper in which they argued that this indicates that quantum mechanics is an incomplete theory. However, no one seriously doubts that the EPR effect is real.

The application of the EPR effect to the neutral B-meson system was first noted by Carter and Sanda. Suppose that one B meson (B_1) decays to a CP eigenstate f at time t_1, and that the second B meson (B_2) decays at time t_2 to a state $g \neq \bar{g}$ (such as $B \to l^\pm \nu X$) that allows us to determine whether it was a B or $\overline{B}$ at time t_2. Then the combined decay asymmetry

$$A(t_1, t_2) = \frac{\Gamma(B_1 \to f)\Gamma(\overline{B}_2 \to \bar{g}) - \Gamma(B_1 \to f)\Gamma(B_2 \to g)}{\Gamma(B_1 \to f)\Gamma(\overline{B}_2 \to \bar{g}) + \Gamma(B_1 \to f)\Gamma(B_2 \to g)} = \sin 2\varphi \sin(x_1 t_1 \mp x_2 t_2),$$

where the minus sign holds for C(odd) states: $|B_1 \overline{B}_2\rangle - |\overline{B}_1 B_2\rangle$.

If we don't observe the decay times, the integrated asymmetry is

$$A = \frac{x_1 \mp x_2}{(1 + x_1^2)(1 + x_2^2)} \sin 2\varphi,$$

which vanishes for C(odd) states in which $B_1 = B_2$ (*i.e.*, $B_d \overline{B}_d$ or $B_s \overline{B}_s$).

For B_d^0-$\overline{B}_d^0$ produced at the $\Upsilon(4S)$ at an e^+e^- collider, we have only C(odd) states, and hence there will be no signal for CP violation unless one can observe the time evolution. This is the well-known justification for the construction of an asymmetric e^+e^- collider, which is a costly consequence of the EPR effect.

8 Analysis of Time-Resolved Decay Asymmetries

In the previous subsection we noted that our proposed method of analysis of CP violation in the neutral B system would yield a null result if we integrate over time and if the B-$\overline{B}$ pair was produced in a C(odd) state. As the latter condition holds for B's produced at the $\Upsilon(4S)$ resonance at an e^+e^- collider, this analysis would be inappropriate there. A clever alternative procedure has been proposed that maximizes the analyzing power at an e^+e^- collider.

Both B's of a produced B-$\overline{B}$ pair must be observed in a CP analysis. We label B_1 as the (neutral) B that decays to the CP eigenstate f, and B_2 as the (charged or neutral) B that decays to a state $g \neq \bar{g}$. Observation of the latter decay permits us to determine whether B_2 was a particle or antiparticle at the moment of its decay. This procedure is often called 'tagging'.

We can accumulate four time distributions, where one B decays at time t_a and the other at time t_b with $t_a < t_b$:

$$\begin{aligned} I: &\quad \Gamma_{B_1\to f}(t_b)\Gamma_{B_2\to g}(t_a), \\ II: &\quad \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b), \\ III: &\quad \Gamma_{B_1\to f}(t_b)\Gamma_{\bar{B}_2\to \bar{g}}(t_a), \\ IV: &\quad \Gamma_{B_1\to f}(t_a)\Gamma_{\bar{B}_2\to \bar{g}}(t_b). \end{aligned}$$

The four distributions can be combined to form asymmetries in various ways. Let

$$A_1(t_a,t_b) \equiv \frac{II+III-I-IV}{I+II+III+IV}.$$

Another asymmetry is

$$A_2(t_a,t_b) \equiv \frac{III+IV-I-II}{I+II+III+IV}.$$

A third might be defined as

$$A_3(t_a,t_b) \equiv \frac{I+III-II-IV}{I+II+III+IV}.$$

For the case that mesons 1 and 2 are of the same type the four time distributions take the form

$$\begin{aligned} \Gamma_I(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 \pm \sin 2\varphi \sin x(t_a \pm t_b)], \\ \Gamma_{II}(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 + \sin 2\varphi \sin x(t_a \pm t_b)], \\ \Gamma_{III}(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 \mp \sin 2\varphi \sin x(t_a \pm t_b)], \\ \Gamma_{IV}(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 - \sin 2\varphi \sin x(t_a \pm t_b)], \end{aligned}$$

where φ is the CP-violating phase in the decay amplitude for $B_1 \to f$, $x = \Delta M/\Gamma$ is the mixing parameter for neutral B-meson, and the lower sign in the distributions holds for C(odd) states $|B_1\rangle|\overline{B}_2\rangle - |\overline{B}_1\rangle|B_2\rangle$. In the above, time is measured in units of the lifetime, $1/\Gamma$.

Inserting the time distributions into the forms for the asymmetries we have

$$\begin{aligned} A_1 &= \begin{cases} \sin 2\varphi \sin x(t_a - t_b) & C(\text{odd}) \\ 0 & C(\text{even}) \end{cases}, \\ A_2 &= \begin{cases} 0 & C(\text{odd}) \\ \sin 2\varphi \sin x(t_a + t_b) & C(\text{even}) \end{cases}, \\ A_3 &= 0. \end{aligned}$$

Clearly the asymmetry A_1 will be useful at an e^+e^- collider where only C(odd) states are produced.

In B_d decays where $x_d \approx 0.7$ there are about nine lifetimes per oscillation, and so a time-resolved analysis is actually little different than a time-integrated one. Hence it is relevant to consider the time-integrated forms of the asymmetries.

Because of the time ordering in the definition of the distributions *I*-*IV*, the form of the integrals is

$$\int_0^\infty dt_a \int_{t_a}^\infty dt_b \Gamma_I(t_a, t_b),$$

etc. On evaluating these integrals for the case that meson B_1 is of the same type as B_2, we find

$$A_1 = \begin{cases} x \sin 2\varphi/(1+x^2) & C(\text{odd}) \\ 0 & C(\text{even}) \end{cases}.$$

while

$$A_2 = \begin{cases} 0 & C(\text{odd}) \\ 2x \sin 2\varphi/(1+x^2)^2 & C(\text{even}) \end{cases},$$

At a hadron collider the B-$\overline{B}$ pairs are produced in C(even) and C(odd) states with equal probability, so the question arises as to which asymmetry is to be preferred to attain maximum sensitivity to the CP-violating factor $\sin 2\varphi$. Note that the nonzero cases of the asymmetries are affected by the dilution due to mixing in different ways:

$$A_1(C(\text{even})) = \frac{2}{1+x^2} A_2(C(\text{odd})).$$

For the case of B_d-$\overline{B}_d$ production where $x_d \approx 0.7$, the factor $2/(1+x^2) \approx 4/3$, so asymmetry A_1 is slightly to be preferred over A_2.

However, at a hadron collider a B_d meson can be produced along with any of a $\overline{B}_u$, $\overline{B}_d$, or $\overline{B}_s$. Table 3 lists the coefficients K of $\sin 2\varphi$ for the various possibilities of B-$\overline{B}$ production for the two asymmetries. On weighting by the relative production rates we estimate that A_1 is about 1.5 times as large as A_2 at a hadron collider, so clearly should be used.

Table 3: The coefficient K in time-integrated CP-violating asymmetries of the form $A = K \sin 2\varphi$ for various possibilities for B_d-$\overline{B}$ production at a hadron collider. The Weighted coefficient is obtained supposing $x_d = 0.7$, $x_s \gg x_d$, and that $\overline{B}_u$, $\overline{B}_d$, and $\overline{B}_s$ mesons are produced along with a B_d in the proportion 0.375 : 0.375 : 0.25. We have assumed that the lifetimes of all three flavors of B mesons are the same.

Asymmetry	B_d-$\overline{B}_d$	B_d-$\overline{B}_u$	B_d-$\overline{B}_s$	Weighted
A_1	$\frac{x_d}{(1+x_d^2)^2}$	$\frac{x_d}{1+x_d^2}$	$\frac{x_d}{1+x_d^2}\frac{1}{x_s^2}$	≈ 0.25
A_2	$\frac{x_d}{2(1+x_d^2)}$	$\frac{x_d}{2(1+x_d^2)}\frac{1-x_d^2/2}{1+x_d^2/4}$	$\frac{x_d}{1+x_d^2}\frac{1}{x_s^2}$	≈ 0.16

At hadron colliders one typically considers the asymmetry

$$A(t_a, t_b) = \frac{\Gamma_{B_1\to f}(t_a)\Gamma_{\overline{B}_2\to \bar{g}}(t_b) - \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b)}{\Gamma_{B_1\to f}(t_a)\Gamma_{\overline{B}_2\to \bar{g}}(t_b) + \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b)} = \sin 2\varphi \sin(x_1 t_a \pm x_2 t_b),$$

where there was no restriction on t_a and t_b, and the minus sign holds for C(odd) states. This asymmetry is not quite the same as A_1 or A_2, but the time integrated version of this is identical to the time integrated version of A_1. That is, in the time integrated version of A_1 we effectively lose sight of the time ordering of t_a and t_b.

For a final comparison, the coefficient K that holds for use of asymmetry A_2 at an e^+e^- collider is 0.5. This means that the average dilution due to mixing at an e^+e^- collider is one half of that at a hadron collider. Equivalently, we will need four times as many tagged, reconstructed B_d-$\overline{B}$ decays at a hadron collider as at an e^+e^- collider to achieve the same sensitivity to $\sin 2\varphi$. Stated yet another way, the smallest value of $\sin 2\varphi$ that can be resolved to three standard deviations with N events at a hadron collider is $12/\sqrt{N}$, while at an e^+e^- collider this would be $6/\sqrt{N}$.