# PION AND NUCLEON STRUCTURE AS PROBED IN THE REACTION $\pi^{\pm}N \rightarrow \mu^{+}\mu^{-}X$ AT 253 GeV

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#### Abstract

New results are presented from Fermilab experiment E615, in which hadroproduction of muon pairs allows a determination of the quark structure of the initial-state hadrons in the context of a Drell-Yan model.

- Comparision of muon-pair production by  $\pi^+$  and  $\pi^-$  beams shows the cross-section ratio follows Drell-Yan model expectations, except for a dip in  $\sigma(\pi^+)/\sigma(\pi^-)$  near  $x_{\pi}$  near 1.
- The same data are also used to extract the ratio of the sea to valence quark distributions in the nucleon, with improved accuracy for  $x_N < 0.1$  compared to deep-inelastic neutrino scattering.
- The Drell-Yan analysis of continuum muon-pair production has been extended down to a mass of 3.2 GeV/c<sup>2</sup> for the  $\pi^-$  data sample. This provides stronger evidence of a rise in the nucleon structure function for  $x_N < 0.06$  compared to that extracted in deep-inelastic lepton scattering.
- The issue is raised of the nonuniqueness of the definition of  $x_{\pi}$  and  $x_N$  used in the Drell-Yan analysis. A definition proposed by Soper has superior invariance properties to that commonly used. An analysis based on this definition yields generally similar results for the pion and nucleon structure function, compared to use of the common definition. However, the pion structure function shows a larger intercept at  $x_{\pi} = 1$  when the definition of Soper is used.

#### Introduction

The possibility of probing the structure of hadrons via the production of lepton pairs in hadronic collisions was first pointed out by Drell and Yan.<sup>1]</sup> The well-known hypothesis is that the muon pair materializes from a virtual photon created in the annihilation of a quark from one initial hadron with an antiquark from the other hadron. This effect probes the quark structure of both the beam and target hadrons, and thereby allows a determination of the structure of any long-lived hadron. In a precursor to the present experiment, our group made the first measurement of the pion structure function.<sup>2</sup>

Our previous experiment also provided evidence<sup>3]</sup> that as the momentum of the annihilating antiquark in the pion approaches that of the pion  $(x_{\pi} \rightarrow 1)$ , the virtual photon becomes longitudinally polarized. This result had been predicted by Berger and Brodsky,<sup>4]</sup> in a QCD calculation that takes note of nonzero transverse momentum of the constituent quarks. This calculation also predicted a nonzero intercept to the quark structure function of the pion at  $x_{\pi} = 1$ , which effect could not be explored in our earlier experiment. Fermilab experiment E615<sup>5</sup>] was performed to explore the forward production of muon pairs in pion-nucleon collisions. The apparatus is sketched in fig. 1 and has been described in detail elsewhere.<sup>6</sup>]. Data were collected with 80- and 253-GeV pion beams, and the main analyses to determine the hadron structure functions have been published.<sup>7,8</sup>] The present paper is concerned with a recent analysis by J.G. Heinrich<sup>9</sup>] of data from both  $\pi^+$  and  $\pi^$ beams, and an analysis by the author of continuum muon-pair production down to a mass of  $3.2 \text{ GeV/c}^2$ . These new results are presented in secs. 1 and 2, respectively, and the Appendix reviews the interesting controversy over the definition of the kinematic variables used in the Drell-Yan analysis.



# E 615 APPARATUS

FIG. 1. The E 615 apparatus.

# 1. Comparision of Muon-Pair Production by $\pi^+$ and $\pi^-$ Beams at 253 GeV

In this comparison two physics issues can be addressed:

- Is the continuum production of muon pairs consistent with the Drell-Yan model?
- Then, accepting the validity of the Drell-Yan model, the ratio of the sea- to valence-quark distributions in the nucleon can be determined.

Our 253-GeV pion beams were derived from interactions of an 800-GeV proton beam with a Be target. Because the pion flux was in excess of  $10^8$ /sec the composition of the 'pion' beams was not directly measured, but inferred from measurements of a 400-GeV proton beam on a Be target.<sup>10</sup>]

Beam Charge	$\pi$ (%)	K (%)	p (%)	
_	$92.8 \pm 1.1$	$5.5\pm0.1$	$1.7 \pm 0.1$	
+	$53.8 \pm 0.7$	4.1 ± 0.1	$42.1 \pm 0.6$	

Table I. Composition of the 253-GeV pions beams.

In obtaining the cross section for muon-pair production by pions, we make a correction for production by kaons and protons using measurements by the NA3 collaboration<sup>11]</sup> for the latter two types of particles. The largest correction is for production by protons, which, however, dies out rapidly with  $x_{\pi}$ . The analysis is limited to the region  $x_{\pi} > 0.36$ , and the cross section for muon-pair production by protons is 10% of that by  $\pi^+$  at  $x_{\pi} = 0.36$ . (In this section we use the old definitions, (A4), of  $x_{\pi}$  and  $x_N$ , as discussed in the Appendix.)

To avoid contamination due to muon pairs from the decay of  $J/\psi$ ,  $\psi'$  and  $\Upsilon$  resonances, only pairs with  $4.05 < M_{\mu^+\mu^-} < 8.55 \text{ GeV/c}^2$  were used in the analysis.

Figure 2 shows the ratio of cross sections thus measured, where

$$rac{\sigma(\pi^+)}{\sigma(\pi^-)}\equiv rac{\sigma(\pi^+N o\mu^+\mu^-X)}{\sigma(\pi^-N o\mu^+\mu^-X)}.$$

To compare the observed ratio with the Drell-Yan model we use the relation

$$\frac{\sigma(\pi^+)}{\sigma(\pi^-)} = \frac{V_{\pi}(x_{\pi})G_N^+(X_N) + S_{\pi}(x_{\pi})H_N(x_N)}{V_{\pi}(x_{\pi})G_N^-(X_N) + S_{\pi}(x_{\pi})H_N(x_N)},$$

where

$$G^{+} \equiv \frac{1}{9} \left[ \left( 1 - \frac{Z}{A} \right) V_{p}^{u} + \left( \frac{Z}{A} \right) V_{p}^{d} + 5S_{p} \right]$$

$$G^{-} \equiv \frac{1}{9} \left[ \left( 4\frac{Z}{A} \right) V_{p}^{u} + 4 \left( 1 - \frac{Z}{A} \right) V_{p}^{d} + 5S_{p} \right]$$

$$H \equiv \frac{1}{9} \left[ \left( 1 + 3\frac{Z}{A} \right) V_{p}^{u} + \left( 4 - 3\frac{Z}{A} \right) V_{p}^{d} + 11S_{p} \right]$$

Here Z is the number of protons in the (tungsten) target nucleus, and A is the number of nucleons in the nucleus.

The pion valence structure function,  $V_{\pi}$ , is taken from our determination of this from the  $\pi^-$  data alone,<sup>8]</sup> while the pion sea function,  $S_{\pi}$ , can be neglected for  $x_{\pi} > 0.36$ . The nucleon quark distributions are taken from the so-called second set of the complication of



**FIG. 2.** The ratio of cross sections for muon-pair production by  $\pi^+$  and  $\pi^-$  beams at 253 GeV as a function of  $x_{\pi}$ . The restrictions on  $x_N$  derive from cuts on the pair mass to avoid muons from resonance decay. The smooth curves are Drell-Yan model predictions using the pion structure function from analysis of our  $\pi^-$  data <sup>8</sup>] and nucleon structure functions from Duke Owens <sup>12</sup>] (second set).

Duke and Owens.<sup>12]</sup> Recall that in the Drell-Yan model the ratio  $\sigma(\pi^+)/\sigma(\pi^-)$  would be 1/4 if we could ignore sea quarks, and if Z = A/2.

On the whole, the agreement with the Drell-Yan model is excellent, with the exception of the point at  $x_{\pi} = 0.96$  which is about 2.5 standard deviations below the model.

If we accept fig. 2 as evidence that the Drell-Yan model applies to that data, we can extract the proton sea-quark distribution,  $S_p$ . As we wish to avoid the use of proton valencequark distributions measured in other experiments, we report only the ratio

$$\frac{S_p(x_N)}{V_p^u(x_N)+V_p^d(x_N)}.$$

Our determination of this ratio is plotted as a function of  $x_N$  in fig. 3. For comparison the experimental results of the CDHS collaboration<sup>13]</sup> are also shown; they studied deepinelastic scattering with  $\nu_{\mu}$  and  $\bar{\nu}_{\mu}$  beams. Our data provide an improved measure of the sea-quark distribution in the region  $x_N < 0.1$ .



**FIG. 3.** The ratio of the sea-quark to the valence-quark distribution in the proton. Our results were obtained with the cuts  $4.05 < M_{\mu\mu} < 8.55$  GeV /  $c^2$  and  $x_{\pi} > 0.36$ . The triangles show the results from the CDHS collaboration <sup>13]</sup> and the curves are from EHLQ <sup>14]</sup> and Duke-Owens <sup>12]</sup> parametrizations.

2. Analysis of the  $\pi^-$  Data for 3.2.  $< M_{\mu\mu} < 8.55 \text{ GeV/c}^2$ .

At the completion of the main structure function analyses of E615,<sup>7,8]</sup> a provocative feature remained in the measured nucleon structure function, as shown in fig. 4. At low values of  $x_N$ , the nucleon structure function obtained in the Drell-Yan analysis rises above that found in deep-inelastic scattering. This trend is possibly seen in another experiment, NA3 at CERN,<sup>15]</sup> which shows an effect at low  $x_N$  at 150 Gev (see fig. 4), and at 200 GeV, but not at 280 GeV.

In our analysis,<sup>8]</sup> the lower limit of  $x_N = 0.04$  was set by the requirement that  $M_{\mu\mu} > 4.05$  GeV/c<sup>2</sup>, to avoid contamination from the  $J/\psi$  and  $\psi'$  vector mesons. However, we have a large sample of muon pairs with masses extending to below 3.0 GeV/c<sup>2</sup> that could shed light on the low- $x_N$  issue if the vector-meson signal were subtracted. Here we give a preliminary report of such an analysis.

The new analysis also includes two other features:

- No attempt is made to correct for Fermi motion in the nuclear target, in conformity with the now-standard practice in structure-function analysis of deep-inelastic-scattering experiments;
- The analysis is made using both the usual definitions (A4) for  $x_{\pi}$  and  $x_N$ , as well as the frame-independent definition of Soper (A7). The latter is conceptually preferable, as reviewed in the Appendix, and should make some difference in the analysis near  $x_{\pi} = 1$ .



**FIG. 4.** The nucleon structure function  $G_N^-(x_N)$  as measured in  $\pi^- N \rightarrow \mu^+ \mu^- X$  at 80<sup>-7</sup> [150<sup>-15]</sup> and 253<sup>-8</sup> GeV. The curve is from the Duke-Owens nucleon structure functions <sup>12]</sup>. The three sets of measured points have been normalized to the curve at  $x_N = 0.25$ .

To subtract the effect of the vector mesons, the cross sections were binned on a grid of  $x_{\pi} \cdot x_N$ . The bin size in  $x_{\pi}$  was 0.02, and the bin size for  $x_N$  was 0.01 for  $x_N < 0.16$  and 0.02 for  $x_N > 0.16$ . According to (A3), for a slice at fixed  $x_{\pi}$ ,  $x_N$  is proportional to  $M^2$ , which permits a resonance subtraction to be made at each slice of  $x_{\pi}$ . For this the  $x_N$  bins were subdivded into smaller intervals whose width corresponded to about 50 MeV/c<sup>2</sup> in mass. The subtraction was greatest at low  $x_{\pi}$ , and was almost negligible near  $x_{\pi} = 1$  (the region which corresponds to low  $x_N$  at a given mass). Figure 5 gives an impression of the quality of the subtraction procedure at the extremes of the region of  $x_{\pi}$  used in the analysis.

The subtracted cross sections were then used to determine the pion and nucleon structure functions, using the procedure described in ref. 8. An  $x_{\pi}$ - $x_N$  bin was used only if it was entirely within the specified mass limits. The latter were  $M < 8.55 \text{ GeV/c}^2$  and M > 3.2, 3.6, or 4.0 GeV/c<sup>2</sup>. There are about 36,000 pairs with mass above 4.0 GeV/c<sup>2</sup>, and 70,000 with mass above 3.2 GeV/c<sup>2</sup>.

The pion valence structure function,  $V_{\pi}(x_{\pi})$ , was determined in a fit to the grid of cross sections  $d\sigma/dx_{\pi}dx_N$  assuming the nucleon quark distributions have the form found by the CCFRR collaboration,<sup>16]</sup> and assuming QCD evolution of these distributions as parametrized by Buras and Gaemers.<sup>17]</sup> Following the Berger-Brodsky model<sup>4]</sup> the pion structure function was parametrized as

$$V_{\pi}(x_{\pi}) = x_{\pi}^{lpha}(1-x_{\pi})^{eta} + \gamma rac{2x_{\pi}^{2}}{9M_{\mu\mu}^{2}}.$$

An overall normalization factor, K, was left as a parameter so the the pion and nucleon structure functions could be normalized to 1 when interpreted as probability distributions. (This requires an assumption as to the fraction of the pion's momentum carried by gluons,



**FIG. 5.** Illustration of the J/ $\psi$  and  $\psi'$  resonance substraction procedure at  $x_{\pi} = 0.21$  and 0.99. The corrected continuum signal for  $M^2 > 10.32 \text{ GeV}^2 / c^4$  was used in the structure-function determination.

taken to be 0.47.) Table II shows the results of the fitting procedure for three lower mass bounds, and two definitions of the  $x_{\pi}$ .

The intercept  $\gamma$  of  $V_{\pi}$  at  $x_{\pi} = 1$  appears to grow more pronounced as the minimum-mass cut is lowered. The use of the frame-independent definitions (A7) also tends to enhance the intercept, as was anticipated. In the Berger-Brodsky model, parameter  $\beta$  should be 2; the use of (A7) moves  $\beta$  in that direction.

$\begin{array}{c} Minimum\\ Mass\\ (GeV/c^2) \end{array}$	3.2	3.6	4.0	3.2	3.6	4.0
$x_i$ Definition	(A4)	(A4)	(A4)	(A7)	(A7)	(A7)
K	$2.27 \pm 0.01$	$2.48\pm0.03$	$2.28\pm0.04$	$2.07~\pm~0.01$	$2.43 \pm 0.03$	$2.02 \pm 0.04$
$\alpha$	$0.54\pm0.01$	$0.49\pm0.01$	$0.50\pm0.01$	$0.62~\pm~0.01$	$0.49\pm0.01$	$0.58\pm0.02$
eta	$1.28\pm0.01$	$1.27\pm0.01$	$1.22\pm0.02$	$1.47~\pm~0.01$	$1.35\pm0.01$	$1.39\pm0.02$
$\gamma$	$1.49\pm0.04$	$1.38\pm0.05$	$1.09\pm0.10$	$2.01~\pm~0.02$	$1.84\pm0.04$	$1.71\pm0.05$

Table II. Parameters from the fit for the pion structure function.

Another type of structure-function analysis is to project the cross section  $d\sigma/dx_{\pi}dx_N$ onto either the  $x_{\pi}$  or  $x_N$  axis, using a functional form to integrate over the structure function of the variable thus suppressed (see ref. 8 for details). When projecting out the pion structure function  $V_{\pi}$  we used the CCFRR quark distributions<sup>16</sup>] at  $Q^2 = 27.7 \text{ GeV}^2/c^4$ . When projecting out the nucleon structure function  $G_N$  we use the pion structure function as found in the fitting procedure just described. Results are shown in fig. 6 for various minimum-mass cuts, with the frame independent definition of the  $x_i$ .

The pion structure function, shown in fig. 6a, clearly appears to have a finite intercept at  $x_{\pi} = 1$ . The increasing statistical evidence for this as the minimum-mass cut is lowered can be seen in Table II.

Figure 6d indicates the nucleon structure function for a minimum-mass cut of 4.0 GeV/c<sup>2</sup>, as used in our previous analysis. The sharp rise at very low  $x_N$  is evident. As the minimum-mass cut is lowered (figs. 6b-c) the rise becomes a broader structure. The low- $x_N$  effect seems unlikely to be a simple artifact of the mass cut, but its physical significance remains unclear.

#### Appendix: The Battle of the x's.

Although the concept of the Drell-Yan analysis has been in existence for 18 years there remains a small controversy as to the proper definition of the kinematic variables to be used at finite laboratory energies. Here we review four possibilities presently under discussion:

- A. The commonly used definition of  $x_{\pi}$  and  $x_N$ ;
- B. A frame-independent definition due to Soper;
- C. A light-cone-variable definition;
- D. A definition proposed by Berger which includes effects of the 'spectator' quark in the beam pion.

We wish to analyze the Drell-Yan reaction

$$p_1+p_2 \to M+X,$$

where M is the virtual photon which materializes as a muon pair;  $p_1$  and  $p_2$  label the initialstate hadrons, and throughout this Appendix the particle labels are also taken to represent the particle 4-vectors. We think of this reaction as arising from the quark-antiquark annihilation:

$$q_1 + q_2 \to M. \tag{A1}$$



**FIG. 6. a)** The pion structure function  $V_{\pi}(x_{\pi})$  obtained by the projection method for  $M_{\mu\mu} > 3.2~GeV / c^2$ , using nucleon quark distributions from the CCFRR collaboration  $^{16]}$ . **b)-d)** The nucleon structure function  $G^-_N$  ( $x_N)$ ) obtained by the projection method for various minimum-mass cuts, using our fitted form for the pion structure function.

The quark 4-vectors are related to the hadron 4-vectors by the famous  $x_i$ 's:

$$q_i = x_i p_i. \tag{A2}$$

That is,  $x_i$  is a measure of the fraction of the momentum of the parent hadron that is carried by a quark. It is the precise definition of the  $x_i$  that is in question.

Now

$$M^2 = q_1^2 + q_2^2 + 2q_1 \cdot q_2 = x_1 x_2 (2p_1 \cdot p_2),$$

where we suppose that  $q_i^2 = 0$ . Thus we arrive at the first relation among the  $x_i$ :

$$x_1 x_2 = \frac{M^2}{2p_1 \cdot p_2}.$$
 (A3)

This relation is expected to hold so long as the quark masses may be taken as zero, which is not the case as  $x_{\pi} \to 1$  in the Berger-Brodsky model.<sup>4</sup>

### A. The Commonly Used Definitions.

The usual conventions for the  $x_i$  derive from an emphasis on the center-of-mass frame of the colliding hadrons. One introduces the Feynman-x of the muon pair:

$$x_F = \frac{2P_L^{\star}}{\sqrt{s}},$$

where  $P_L^*$  is the longitudinal momentum (along the beam axis) of the muon pair in the centerof-mass frame of the initial hadrons. Then one supposes that  $x_F$  is related to the quark  $x_i$  by

$$x_F = x_1 - x_2.$$

Combining this with (A3), and approximating

$$s=(p_1+p_2)^2\approx 2p_1\cdot p_2,$$

yields the usual form:

$$x_{1,2} \equiv \frac{\pm x_F + \sqrt{x_F^2 + 4M^2/s}}{2}.$$
 (A4)

Although (A3) appears in the original paper of Drell and Yan,<sup>1]</sup> the first publication of (A4) of which I am aware was by Farrar.<sup>18]</sup> Other early definitions<sup>19,20]</sup> appear not to have gone into general use.

In practice, the quark  $x_i$  must be related to quantities in the laboratory frame. In the case of fixed-target experiments the form (A4) has a conceptually unpleasant feature. To see this we introduce the laboratory four-vectors

$$p_{1} \approx (E_{1}, 0, 0, E_{1}) \qquad (\text{relativistic beam}); \\p_{2} = (M_{2}, 0, 0, 0) \qquad (\text{fixed target}); \qquad (A5) \\M = (E, P_{T}, 0, P_{L}).$$

With these we have  $P_L^{\star} = \gamma P_L - \gamma \beta E$ , where  $\gamma = (E_1 + M_2)/\sqrt{s}$ , and  $\gamma \beta = E_1/\sqrt{s}$ . We may approximate

$$P_L \approx E - rac{M_T^2}{2E}$$
 where  $M_T^2 \equiv M^2 + P_T^2$ . (A6)

Then we find

$$x_1 pprox rac{E}{E_1} - rac{P_T^2}{2M_2E}$$

Thus in the usual definition, the momentum fraction of a quark in hadron 1 depends on the mass of hadron 2.

#### B. The Frame-Independend Definition of Soper.

During the St. Croix Workshop in Oct. 1987, D. Soper proposed a definition which is a frame-independent version of that given in a paper by him.<sup>21</sup>

The motivation is to accept (A3) as valid, and then to express the  $x_i$  only in terms of the 4-vector products  $p_1 \cdot M$ ,  $p_2 \cdot M$  and  $p_1 \cdot p_2$  to maintain frame independence. If one imposes a requirement of the exchange symmetry  $x_2(p_1, p_2) = x_1(p_2, p_1)$ , then the essentially unique definition is

$$x_{1} \equiv \sqrt{\frac{M^{2}}{2p_{1} \cdot p_{2}}} \frac{p_{2} \cdot M}{p_{1} \cdot M}; \qquad x_{2} \equiv \sqrt{\frac{M^{2}}{2p_{1} \cdot p_{2}}} \frac{p_{1} \cdot M}{p_{2} \cdot M}.$$
(A7)

On evaluating  $x_1$  using the lab-frame 4-vectors (A5) we have

$$oldsymbol{x}_1 = rac{E}{E_1}rac{M}{M_T}\sqrt{rac{E+P_L}{2E}} pprox rac{E}{E_1}\left(1-rac{P_T^2}{2M^2}
ight),$$

where we have used the approximation (A6).

Soper's definition of  $x_1$  is free of any dependence on  $M_2$ , and will typically take on a smaller numerical value than the usual definition (A4).

C. A Light-Cone-Variable Definition.

Another fact of life in the laboratory is that the muon pair emerges with nonzero transverse momentum. This is not necessarily consistent with the relation (A2), which implies that the quarks have no transverse momentum.

One way of dealing with the uncertainty in the origin of the muon-pair transverse momentum is to require that the definition of the  $x_i$  be invariant only under transformations which keep constant the value of the transverse momentum. The class of such transformation consists of boosts along the beam direction, and leads to the use of light-cone variables.

Taking axis 3 as the beam axis, one defines the light-cone variables

$$a^+\equiv a_0+a_3 \quad ext{ and } \quad a^-\equiv a_0-a_3,$$

for a 4-vector  $a = (a_0, a_1, a_2, a_3)$ . Then for a boost along axis 3 described by the 4-vector  $b = (\gamma, 0, 0, \gamma\beta)$ , the transformed components of a are just

$$a'^+ = b^+ a^+$$
 and  $a'^- = b^- a^-$ .

If hadron 1 moves along the + direction of axis 3, then a massless quark inside it would have  $q_1^- = 0$ , while a massless quark in hadron 2 would have  $q_2^+ = 0$ . A natural way of implementing the relations (A2) in terms of light-cone variables is

$$x_1 \equiv rac{q_1^+}{p_1^+}; \qquad x_2 \equiv rac{q_2^-}{p_2^-}$$

To express this in terms of measurable quantities, note that (A1) implies  $M^+ = q_1^+$  and  $M^- = q_2^-$ . Hence one might define

$$x_1 \equiv \frac{M^+}{p_1^+}$$
 and  $x_2 \equiv \frac{M^-}{p_2^-}$ . (A8)

However, this definition is not consistent with (A3) for now we have

$$x_1x_2=\frac{M_T^2}{s}.$$
 (A9)

In principle, one might accept (A9) as a consequence of accomodating nonzero transverse momenta. But in practice, (A9) leads to severe difficulties in an analysis of muon-pair production at masses near to those of vector mesons, such as the  $J/\psi$  or  $\Upsilon$ , which decay to muon pairs. While the vector mesons have a definite mass, they have a broad spectrum in  $M_T$ , which may overwhelm the Drell-Yan pairs.

It is still consistent with the idea of invariance under boosts along axis 3 to define

$$x_1 \equiv \frac{M^+}{p_1^+} \frac{M}{M_T}$$
 and  $x_2 \equiv \frac{M^-}{p_2^-} \frac{M}{M_T}$ , (A10)

which restores (A3), and which could be implemented in practice. When evaluated in the lab frame using (A5), we have

$$x_1 = rac{E}{E_1}rac{M}{M_T}rac{E+P_L}{2E} pprox rac{E}{E_1}\left(1-rac{P_T^2}{2M^2}
ight).$$

Hence there is no practical difference between the light-cone definition (A10) and the frameindependent definition (A7).

#### D. A Definition Proposed by Berger.

In the Berger-Brodsky model<sup>4</sup>] the relation (A1) is replaced by

$$p_1 + q_2 \to M + q_f, \tag{A11}$$

where  $q_f$  is the final state of the spectator quark in hadron 1. E. Berger (private communication) has suggested that (A11) leads to the definition

$$x_1 \equiv \frac{M^+}{p_1^+}$$
 and  $x_2 \equiv \frac{M^- + q_f^-}{p_2^-}$ . (A12)

On evaluating this with the lab-frame quantities (A5) he obtains

$$x_1 = rac{E+P_L}{2E_1}$$
 and  $x_2 = rac{E-P_L}{M_2} + rac{P_T^2}{2E_1M_2(1-x_1)}$ 

For this, (A3) is replaced by  $x_1x_2s = M_T^2 + \cdots$ , which makes Berger's definition difficult to implement for masses near that of the  $J/\psi$  meson. The definitions (A12) are well-behaved in the sense that  $x_1$  does not depend on  $M_2$ , but  $x_2$  takes on considerably larger numerical values than with any previous definition.

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The definitions of the  $x_i$  which are readily implemented in practice are (A4), (A7), and (A10). Of these, the usual one, (A4), is somewhat unfortunate, and either (A7) or the nearly identical (A10) would be preferable in future analyses.

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