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Effects of Transition Radiation and Scintillation on the Synchrotron-Čerenkov Experiment

1. Transition Radiation

X.K. Maruyama suggested that transition radiation may not be completely negligible in the experiment, and calculations indicate that this is indeed so. However, it remains a small but measurable correction, which will add to the interest of the experiment if it can be cleanly demonstrated.

a. An estimate.

I first give a quick estimate, which will serve as a check for the detailed calculation presented later.

Suppose the relativistic electron beam passes from vacuum into an opaque dieletric medium (the mirror) with a large dielectric constant. We collect the light emitted in vacuum close the the mirror surface (which I take to be perpendicular to the electron's path). Then the angular distribution of radiated photons may be taken from a standard formalism such as the book of Ter-Mikaelian, p. 227, eq. (26.17) to be

$$\frac{dN_{\text{Transition}}}{d\theta} = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\theta^3\left(\frac{1}{\frac{1}{\gamma^2}+\theta^2}\right)^2,$$

where $\gamma = 1/\sqrt{1-\beta^2}$ describes the velocity of the electron, and ω is the frequency of the light. Integrating this from $\theta = 0$ to ~ 1 we find

$$dN_{\text{Transition}} \sim \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\ln\gamma.$$

Note the very slow dependence of the transition radiation on the electron energy in this circumstance.

For comparision we write the Čerenkov radiation rate over path length L as

$$dN_{\text{Cerenkov}} = 2\pi\alpha\frac{L}{\lambda}\sin^2\theta_C\frac{d\omega}{\omega},$$

where λ is the wavelength of the emitted light, and θ_C is the Čerenkov angle. We are interested in the region near Čerenkov threshold where interference exists with synchrotron radiation. That is, $\theta_C \sim 1/\gamma$. In this case,

$$dN_{\text{Cerenkov}} \sim 2\pi\alpha\frac{L}{\lambda}\frac{1}{\gamma^2}\frac{d\omega}{\omega}.$$

Hence

$$\frac{dN_{\text{Transition}}}{dN_{\text{Cerenkov}}} \sim \frac{1}{\pi^2}\frac{\lambda}{L}\gamma^2 \ln \gamma.$$

Evaluating this for

$E_e = 700$ MeV, so that $\gamma \sim 1370$;

$\lambda \sim 0.5\ \mu$m (optical);

$L = 5$m,

we find

$$\frac{dN_{\text{Transition}}}{dN_{\text{Cerenkov}}} \sim 0.1.$$

If we actually desire a stronger relative signal of transition radiation we should raise the electron energy. Then we have only a slight increase in the absolute rate of transition radiation, but the amount of Čerenkov radiation in the synchrotron-Čerenkov interference region drops!

b. Detailed calculation.

In practice the transition radiation will not arise in vacuum but in the Helium gas. As the gas pressure increases past the Čerenkov threshold there will be interference between transition radiation and Čerenkov radiation which would be interesting to observe. This possibility has been considered in a theoretical paper: L.L. DeRaad, W.-Y. Tsai and T. Erber, Phys. Rev. **D18**, 2152 (1978). They present their results as a correction term to the usual expression for Čerenkov radiation which includes the combined effects of transition radiation and the interference between transition and Čerenkov radiation.

I first summarize the formalism I used to calculate the effect. The equations numbers refer to those of DeRaad *et al.*

$$dN_{\text{C-Tr}} = \frac{\alpha}{\pi}\frac{d\omega}{\omega}\left(F(\epsilon_1,\epsilon_2) + F(\epsilon_2,\epsilon_1)\right); \qquad (1.4,\ 3.21a,\ 3.21b,\ \&\ 4.9)$$

$$\begin{aligned} F(\epsilon_1,\epsilon_2) = & \frac{-1}{\sqrt{\epsilon_1}} - \frac{\beta^2\epsilon_1{\epsilon_2}^2[\beta^2(\epsilon_1+\epsilon_2)-1]}{X_1X_2\sqrt{\epsilon_1+\epsilon_2}}\mathcal{L}_1(\epsilon_1,\epsilon_2) \\ & + \left(\frac{1}{\epsilon_2} - \beta^2 + \frac{2\epsilon_1\beta^2}{X_2}\right)\mathcal{L}_2(\epsilon_2) \qquad (3.21c) \\ & + \frac{2(1-\beta^2\epsilon_2)}{\beta^2\epsilon_{21}X_2}\left[(2-\beta^2\epsilon_2)\mathcal{L}_2(\epsilon_2) + (2-\beta^2\epsilon_1)\mathcal{L}_3(\epsilon_1,\epsilon_2)\right]; \end{aligned}$$

$$\epsilon_{21} = \epsilon_2(\omega) - \epsilon_1(\omega); \qquad (3.18)$$

$$X_i = \epsilon_1 + \epsilon_2 - \beta^2 \epsilon_i^2, \quad i = 1, 2; \tag{3.20a}$$

$$\mathcal{L}_1(\epsilon_1, \epsilon_2) = 2 \ln \left(\frac{\epsilon_2 + \sqrt{\epsilon_2(\epsilon_1 + \epsilon_2)}}{\epsilon_1 + \sqrt{\epsilon_1(\epsilon_1 + \epsilon_2)}} \right); \tag{3.20b}$$

$$\mathcal{L}_2(\epsilon_2) = \frac{1}{2\beta} \ln \left(\frac{1 + \beta\sqrt{\epsilon_2}}{|1 - \beta\sqrt{\epsilon_2}|} \right); \tag{3.20c}$$

$$\mathcal{L}_3(\epsilon_1, \epsilon_2) = \left\{ \begin{array}{ll} -\frac{\sqrt{\beta^2\epsilon_{21} - 1}}{\beta} \tan^{-1} \left(\frac{\sqrt{\beta^2\epsilon_{21} - 1}}{\beta\sqrt{\epsilon_1}} \right) & \text{if } \beta^2\epsilon_{21} \geq 1; \\ \frac{\sqrt{1 - \beta^2\epsilon_{21}}}{2\beta} \ln \left(\frac{\beta\sqrt{\epsilon_1} + \sqrt{1 - \beta^2\epsilon_{21}}}{|\beta\sqrt{\epsilon_1} - \sqrt{1 - \beta^2\epsilon_{21}}|} \right) & \text{if } \beta^2\epsilon_{21} < 1. \end{array} \right\} \tag{4.8}$$

The terms $\mathcal{L}_2$ and $\mathcal{L}_3$ contain logarithmic singularities at the Čerenkov threshold.

I have evaluated this expression numericallly for 700 MeV electrons in Helium at various pressures which then exit into a medium of dielectric constant $\epsilon = 2.25$. As was done for the synchrotron-Čerenkov effect, I include the effect of phototube efficiency, and consider only one polarization of the radiation. I have ignored the extra complication of possible interference between synchrotron and transition radiation. The results are shown in the figure.

Note that the condition $\theta_C \sim 1/\gamma$ occurs at pressure $\sim$ 0.015 atm., and that the Čerenkov rate there of 0.02 detected photons per electron is indeed about 10 times the vacuum transtion radiation rate of 0.0017 detected photons per electron. In practice it will be hard to resolve the very narrow spike of transition radiation just at Čerenkov threshold, but it may survive as a small bump.

INTERFERENCE EFFECT WITH TRANSITION RADIATION