K.T. McDonald
Princeton U.
June 16, 1993

E-144: Angular Distribution of Synchrotron Radiation

Al Odian and Francesco Villa have pointed out that the angular distribution of synchrotron radiation is broader for lower energy photons. In a calculation described below we find the characteristic angle of 2-eV photons from a 100-Gauss magnet (very soft bends) is about $15/\gamma$, and that of a 667-Gauss magnet (soft bends) is about $30/\gamma$. This radiation is still within the acceptance of the monitor proposed in our note of April 27, 1993.

1 Calculation of the Angular Distribution

We use eq. (14.83) of Jackson, *Classical Electrodynamics, 2nd ed.* (Wiley, 1975):

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3\gamma^2 e^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 \left[(1+\gamma^2\theta^2)^2 K_{2/3}^2(\xi) + \gamma^2\theta^2(1+\gamma^2\theta^2)K_{1/3}^2(\xi)\right],$$

where angle θ is not the usual polar angle but rather is measured with respect to the plane of the orbit. The critical energy is written as

$$\hbar\omega_c = \frac{3}{2}\gamma^2 mc^2 \frac{B}{B_{\rm crit}},$$

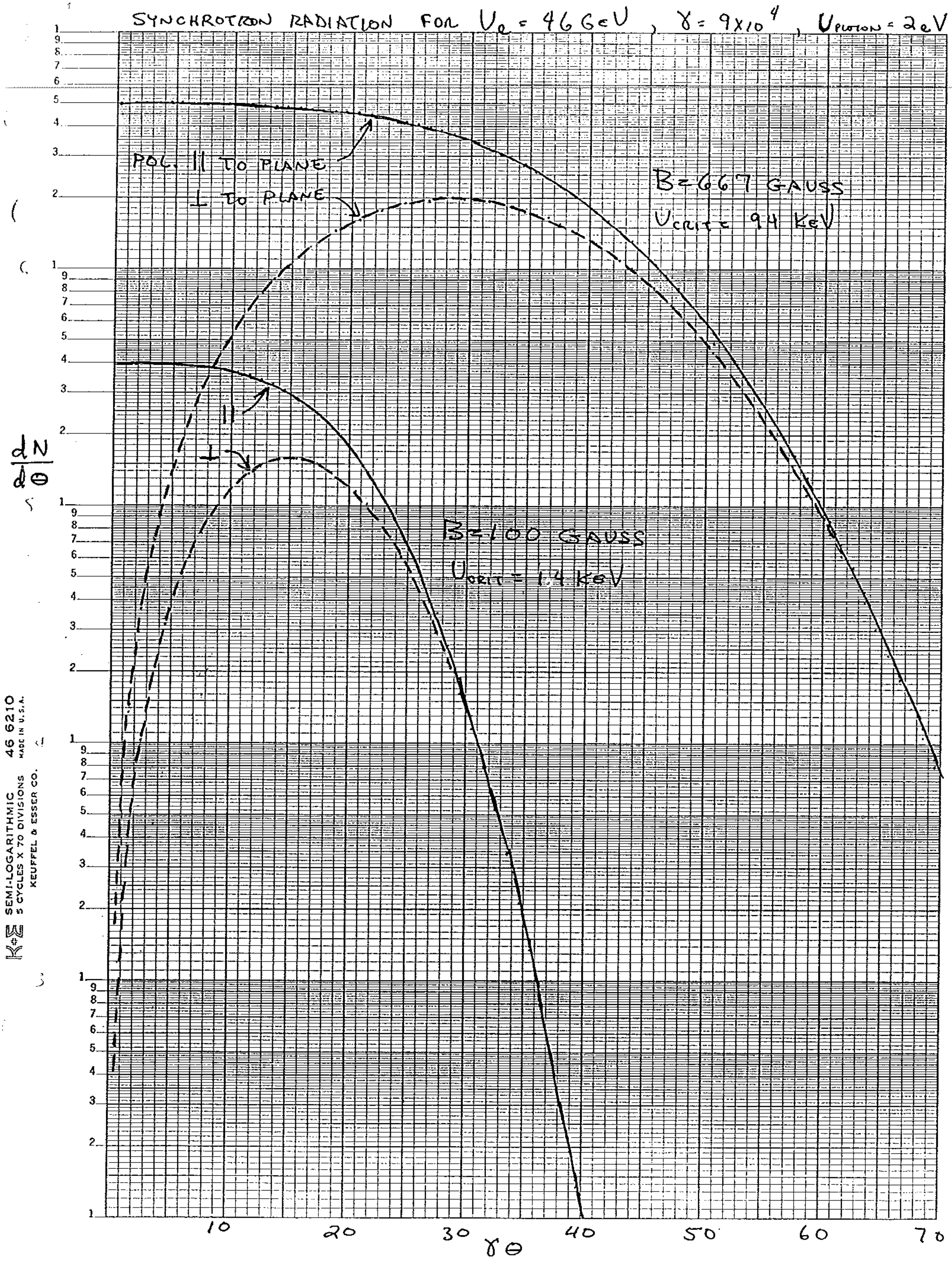
where B is the magnetic field, $B_{\rm crit} = 4.4 \times 10^{13}$ Gauss is the QED critical field strength, and γ is the Lorentz factor of the radiating electron. Note that our definition of ω_c follows Sands and various Russian authors, and is 1/2 that used by Jackson. The argument ξ of the modified Bessel functions K is given by

$$\xi = \frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{3/2}.$$

The first term in the angular distribution corresponds to radiation polarized in the plane of the orbit, while the second term is for perpendicular polarization.

Numerical values of the modified Bessel functions were obtained using function BSKR3 of CERNLIB. Results of the calculation are shown in the Figure. We see that the angular distribution due to the soft bend magnets is 15-30 times broader than the characteristic angle $1/\gamma$ assumed in our previous note.

The integrals of the angular distributions appear to agree with Jackson's eq. (14.93) if we include the factor 3.25 he mentions following eq. (14.95).



SYNCHROTRON RADIATION FOR U_e = 46 GeV , γ = 9×10^4 , U_{PHOTON} = 2 eV
POL. ∥ TO PLANE
⊥ TO PLANE
B = 667 GAUSS
U_{CRIT} = 94 KeV
∥
⊥
B = 100 GAUSS
U_{CRIT} = 14 KeV
dN/dθ
γθ
10
20
30
40
50
60
70
SEMI-LOGARITHMIC 46 6210
5 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

2 Effect on the Synchrotron Radiation Monitor

Fortunately the angular acceptance of the synchrotron radiation monitor is ± 1.25 mrad, corresponding to about $\pm 110/\gamma$ for 46-GeV electrons. Hence essentially all of the radiation from the soft bend magnets is still within the acceptance.

The vertical angular distribution of the radiation is directly related to the vertical angle of the electron, and remains as discussed in our earlier note. The horizontal angular distribution is broader by the factor calculated above.

When analyzing the angular distribution we focus the lens at infinity. This lens has now been chosen to be the Canon 400mm/4.5 lens, which we will use with a $2\times$ adapter giving an effect focal length of 800 mm. Previously we had supposed the horizontal angular distribution was ± 40 μrad, dominated by the angular spread of the electron beam. Now we expect the the 667-Gauss soft bend magnet will have a horizontal angular distribution of ± 300 μrad. The vertical angular spread remains at 500 μrad. The 800-mm lens images this angular distribution onto a region of the Electrim CCD some 20 pixels high by 24 pixels wide, which is still quite adequate. The intensity at the CCD is reduced by about a factor of 1/4 compared to our previous estimate.

Thus the observation of the angular distribution of the synchrotron radiation is little changed in principle from our earlier note.

However, the granularity of the proposed spectral measurement is coarsened because of the larger horizontal angular distribution. We will now obtain only 5-6 bins in frequency.