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PROTOTYPE STUDY OF THE STRAW TUBE PROPORTIONAL CHAMBER

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Abstract

In our "Proposal to the SSC Laboratory for Research and Development of a Straw-Tube Tracking Subsystem" we have raised several issues important for operating a large straw-tube tracking system in the SSC environment. This paper reports on prototype studies of the validity of various models of the gas-gain mechanism, of temperature dependence of gas gain, and of electrostatic stability of a long straw tube and some related problems.

1. Gas-gain study

In designing a gas wire chamber, the gas gain always is one of the most important and most basic considerations. Many experimental studies have been carried out to determine the gas gain of various counters under differing gas conditions, and several gas-gain formulae to fit the experimental data have been proposed.[1–7] It will be useful to identify the theoretical formula which best fits our straw-tube data.

A prototype module of straw tubes has been used for this study. It consists of seven short straw-tube counters, each 7 cm in length and 0.7 cm in diameter. The straw tube itself is made of a two-ply laminate of an inner polycarbonate film about 14 μm thick surrounded by a layer of 12.5-μm Mylar. The polycarbonate film is aluminized on its inner surface to about 1000 Å thickness. The tubes and end plugs were obtained from Ohio State U.[8] The seven tubes have five different anode wire sizes: 0.0203, 0.0254, 0.051, 0.076, and 0.127 mm diameter.

Four kinds of gas mixtures, P-10 [= Ar/CH_4 (90/10)], Ar/C_2H_6 (50/50), Ar/CO_2(50/50) and Ar/CO_2(20/80) have been tested with this prototype thus far. We used an Fe^{55} source and measured the charge out of the test chamber with an Ortec model 142PC preamplifier, followed by an Ortec model 570 spectroscopy amplifier, whose output was digitized by an Ortec model 916 multichannel analyzer. A calibration of the charge out of the chamber per count in the 916 analyzer was obtained with an Ortec model 419 precision pulser (by charging a 2 pf capacitor). The primary ionisation caused by the Fe^{55} is taken to be 220,223,192 and 179 electrons in P-10, Ar/C_2H_6(50/50), Ar/CO_2(50/50) and Ar/CO_2(20/80), respectively. This is based on an average energy loss per ion pair created of 26.8 eV, 26.5 eV, 30.8 eV and 33.0 eV in these four gas mixtures,[9] noting that the x-ray energy is 5.9 keV.

The data on gas gain *vs.* high voltage for Ar/CO_2(50/50) and Ar/Ethane (50/50) are shown in Fig. 1. For gas gains larger than 2.5×10^4, the signal charge due to the 5.9-keV x-rays will exceed 1 pC and the chamber will no longer be in the proportional mode. Therefore we have restricted our studies to gains below this value.

Among various gas-gain formulae, we used three to fit our experimental data, namely those of Diethorn,[2] Aoyama,[6] and Kowalski.[7]

Diethorn's formula is

$$\frac{\ln G}{V/\ln(R_c/R_a)} = \frac{\ln 2}{\Delta V}\left(\ln \frac{V}{PR_a \ln(R_c/R_a)} - \ln K\right). \tag{1}$$

Here ΔV corresponds to the potential difference through which an electron moves between successive ionizing collisions, and K is the minimum value of E/P below which multiplication cannot occur. Throughout this paper, G is the gas gain, V is the voltage applied to the tube, E is the (position dependent) electric field strength, P is the pressure, R_a is the radius of the anode wire, and R_c is the radius of the cathode surface of the straw tube.

Aoyama's formula is

$$\frac{\ln G}{V/\ln(R_c/R_a)} = \exp\left\{-A\left(\frac{V}{NR_a \ln(R_c/R_a)}\right)^{m-1} - \ln[(1-m)V_I]\right\}, \tag{2}$$

where V_I is the effective ionization potential, and A and m are constants characteristic of the gas.

Kowalski's formula is

$$\frac{\ln G}{V/\ln(R_c/R_a)} = \frac{A_1}{d-1}\left(\frac{V}{PR_a \ln(R_c/R_a)}\right)^{d-1} + B_1, \tag{3}$$

where A_1, B_1, and d are constants of the gas.

The data points and fitted lines are shown in Fig. 2 and Fig. 3 for Ar/CO_2 and Ar/Ethane, respectively. The variance used in the fitting for all of three formula is defined in the same way as

$$\text{Variance} = \sum_{i=1}^{n}\left(\frac{\ln(G_{\text{data}})_i}{V_i/\ln(R_c/R_{a,i})} - \frac{\ln(G_{\text{fit}})_i}{V_i/\ln(R_c/R_{a,i})}\right)^2 \Big/ n, \tag{4}$$

so we can directly compare their goodness of fit. The results are summarized in Table 1. For the other two gas mixtures the results of goodness of fit are similar.

Only slight differences exist among the fits using the three models, so any could be used for the range of conditions we are studying.

2. Temperature dependence of the gas gain

Fig. 1(a). Gas gain *vs.* voltage for straw-tube chambers with five different anode-wire diameters, filled with Ar/CO_2(50/50) gas.

Fig. 1(b). Gas gain *vs.* voltage for straw-tube chambers with five different anode-wire diameters, filled with Ar/Ethane (50/50) gas.

Table 1. Variances and parameters of the fitting.

Gas mix	Fit type	Variance	Fitted parameter
Ar/CO_2 (50/50)	Diethorn	7.584×10^{-7}	$\Delta V = 41.3$ eV; $K = 4.736 \times 10^4$ V/cm·atm
	Aoyama	6.298×10^{-7}	$A = 1.376 \times 10^{-7}$; $m = 0.5$, $V_i = 16.8$ V
	Kowalski	6.276×10^{-7}	$A_1 = 0.543 \times 10^{-2}$ $(\text{m}\cdot\text{Pa})^{d-1}\text{V}^{-d}$; $d = 1.228$, $B_1 = -0.05536/\text{V}$
Ar/Ethane (50/50)	Diethorn	2.014×10^{-7}	$\Delta V = 31.58$ eV; $K = 4.84 \times 10^4$ V/cm·atm
	Aoyama	1.141×10^{-7}	$A = 0.1141 \times 10^{-6}$; $m = 0.4942$, $V_i = 12.86$ V
	Kowalski	1.031×10^{-7}	$A_1 = 0.5578 \times 10^{-2}(\text{m}\cdot\text{Pa})^{d-1}\text{V}^{-d}$; $d = 1.277$, $B_1 = -0.05594/\text{V}$

The heat dissipation due to the electron/ion currents in a straw tube that comes within 10 cm of the beams at 10^{32} luminosity at SSC is 1/3 mWatt.[1]0 This will heat up the gas and consequently the gas gain will be changed. In order to keep the gas gain within a desired range, the gas flow rate must be adequate to cool the heat load. But a large flow rate is difficult to accommodate in a compact straw-tube design, so it is important to know how strong is the temperature dependence of the gas gain.

A. Experimental results

Because of an apparent lack of relevant data in the literature, we have placed the test chamber in an oven to make direct measurements of the temperature dependence. We are able to maintain a constant temperature inside of the oven to ±0.5°C, as monitored by a a thermocouple and microvoltmeter. The gas flow rate was reduced to a very low level to insure that the gas temperature inside the chamber was that of the surrounding oven.

Fig. 4 shows the experimental results for the P-10 gas mixture and the 0.0204-mm anode-wire chamber. Those for Ar/Ethane with 0.0204- and 0.127-mm anode-wire chambers are shown in Fig. 5. From these figures we draw the following qualitative conclusions:

1. The gas gain increases with temperature.
2. Different gas mixtures shows different temperature dependences; Ar/Ethane

Fig. 2. Model fits to the gas gain in Ar/CO_2(50/50) gas.

Fig. 3. Model fits to the gas gain in Ar/Ethane (50/50) gas.

Fig. 4. Temperature dependence of the gas gain in Ar/CO_2(50/50) gas with an 0.8-mil-diameter anode wire.

Fig. 5. Temperature dependence of the gas gain in Ar/Ethane (50/50) with 0.8-mil and 5-mil-diameter anode wires.

Fig. 6. The ratio of relative gain change (dG/G) to the relative temperature change (dT/T) as a function of the gas gain in P-10 and Ar/Ethane (50/50) for 0.8-mil and 5-mil diameter anode wires.

(50/50) is about 2.5 times as sensitive to temperature changes as P-10.

3. The temperature dependence of a 5-mil-diameter anode wire is about twice that of a 0.8-mil one;
4. The temperature dependence is stronger at larger gas gain.
5. A characteristic value of $(dG/G)/(dT/T)$ is 5, as for Ar/Ethane (50/50) with an 0.8-mil-diameter anode wire and a gas gain of 10^4. But see Fig. 6 for variations with gas type, gain, and anode-wire diameter.

B. Model interpretation

Among the early works on gas amplification the well-known model proposed by Rose and Korff[1] has been cited by many authors. But this model is inadequate to reproduce the temperature dependence observed by us. The gas-gain formula derived from their model can be written as

$$\ln G = 2\sqrt{\frac{KNR_aV}{\ln(R_c/R_a)}}\left(\sqrt{\frac{V}{V_t}} - 1\right), \tag{5}$$

where N denotes the gas density, V_t is the threshold voltage at which amplification starts to take place, and K is a constant.

The basic assumption of Rose and Korff was that

$$\alpha(r) = \langle N\sigma(U)\rangle$$

where $\alpha(r)$ denotes the Townsend coefficient (defined by $dn/n = -\alpha(r)dr$ where n is the number of ionization electrons), and $\sigma(U)$ is the ionization cross section as a function of the electron energy U. The dependence of α on the electric field E is through the latter's effect on the spectrum of energies U.

Assuming the gas pressure of the counter remains constant while the temperature is changing, it follows that

$$\frac{dN}{N} = -\frac{dT}{T}. \tag{6}$$

The gas density decreases with increasing temperature; therefore α decreases too, according to the model of Rose and Korff, and as a consequence the gas gain decreases. This contrasts with our observation that the gas gain increases with increasing temperature.

Aoyama's model is based on different assumptions. He expressed α as $1/\lambda_r$ – the number of mean free paths per unit length in the field direction – multiplied by the chance of a free path length longer than λ_I – the mean path length for an electron to travel in the field direction to ionize a gas molecule, *i.e.*,

$$\alpha = \frac{1}{\lambda_r}e^{-\lambda_I/\lambda_r}, \tag{7}$$

where $\lambda_I = V_I/E$, and V_I is the effective ionization potential.

We can write

$$\frac{1}{\lambda_r} \approx \frac{1}{\lambda} = N\sigma,$$

since the mean free path along the field direction is approximately the same as the overall mean free path. Then the Townsend coefficient α is approximately given as

$$\alpha = N\sigma \exp(-N\sigma V_I/E). \tag{8}$$

It follows that

$$\frac{d\alpha}{dN} = \sigma(1 - \lambda_I/\lambda_r) \exp(-N\sigma V_I/E). \tag{9}$$

The sign of $d\alpha/dN$ will depend on the ratio of λ_I to λ_r. On differentiating both sides of eq. (2) and taking eq. (6) into consideration we obtain

$$\frac{dG/G}{dT/T} = A(1-m)\ln G \left(\frac{E_a}{N}\right)^{m-1} = A(1-m)\ln G \cdot S_a^{m-1}, \tag{10}$$

where we have introduced $S_a \equiv E_a/P$ as the ratio of the electric field strength at the anode wire to the gas pressure. Similarly from eq. (1) and eq. (3) we get the following predictions according to Diethorn and Kowalski, respectively:

Diethorn's prediction:

$$\frac{dG/G}{dT/T} = \frac{\ln 2}{\Delta V \cdot \ln(R_c/R_a)} V. \tag{11}$$

Kowalski's prediction:

$$\frac{dG/G}{dT/T} = \frac{A_1}{\ln(R_c/R_a)} (S_a^{d-1} \cdot V). \tag{12}$$

Fig. 7 shows the comparison between our experimental data and these model predictions. The general trends are rather well predicted, but none of the models is able to make perfect predictions. The numerical disparities are within a factor of 2, and the experimental data are always below the model predictions, in which the model parameters were fitted using data taken at a constant temperature.

In summary:

- The gas-gain formulae proposed by Diethorn, Aoyama, and by Kowalski are able to describe the general behaviour of temperature dependence, and the predicted values could be safely used as upper limits.
- The most favorable (*i.e.*, smallest) empirical values for $(dG/G)/(dT/T)$ are about half of the predicted values.

3. Electrostatic instability of the straw-tube chamber

Fig. 7. Comparison of experimental data on the temperature dependence of the gas gain with several models whose parameter were determined at a fixed temperature.

Our goal is to build a large straw-tube tracking system. The length of each straw tube will be up to 2 m. It will greatly simplify the construction process and reduce the inefficient zone of the straw tube chamber if an extra support in the middle of the anode wire can be avoided.

Cylindrical wire chambers are prone to an electrostatic instability in which the anode wire is pulled toward the cathode if the fields are strong enough. A general formula for the highest reachable voltage of stable operation of a straw tube chamber has been derived:[11]

$$V < \sqrt{\frac{T}{2\pi\epsilon_0}\frac{\pi R_c}{L}} \ln\left(\frac{R_c}{R_a}\right) \equiv V_0, \tag{13}$$

where T is the tension of anode wire (Newtons), L is the length of wire (m), and $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. We will generalize this below.

We have studied the accuracy of eq. (13) with a 2-m-long tube, as sketched in Fig. 8. A 6.35-mm inner-diameter, 2-m-long stainless steel tube was fixed on a vertical unistrut channel by five adjustable rod-end bearings. By carefully adjusting the positions of these bearings we were able to secure the straightness of the tube to high accuracy. In the middle of the tube four 2.38-mm-diameter holes were drilled, equally spaced in 90° intervals. Two He-Ne laser beams and a microscope eyepiece were employed to project the image of the hole and anode wire onto a screen from two orthorgonal directions as shown on the left of Fig. 8. The image of a 2.38-mm hole set the scale that was used to estimate the deviation of anode wire from its central location. The image of the hole and anode wire on the screen is sketched in Fig. 9. Actually the image of anode wire is a diffraction pattern, so it appears wider than its geometrical image.

A sample data set of these measurements is given in Table 2. The scan was made of the anode-wire displacement as a function of voltage.

Observation shows that as the voltage is raised the anode wire experiences small but stable displacements, which go over into a large vibration at higher voltage. For practical purposes that latter conditions are unstable, and their onset defines the maximum operating voltage of the chamber.

The instability is aggravated if the anode wire is not concentric with the cathode tube. Here we extend that model of the instability to include this effect.

The capacitance per unit length of the straw-tube chamber as two cylinders not necessarily concentric is[12]

$$C = 2\pi\epsilon_0 \left[\cosh^{-1}\left(\frac{R_c^2 + R_a^2 - D^2}{2R_cR_a}\right)\right]^{-1}.$$

The definitions of R_a, R_c, and the anode-wire displacement D are illustrated in Fig. 10.

When voltage V is applied between the cylinders the potential energy is

$$W = \frac{1}{2}CV^2.$$

Fig. 8. Sketch of the setup to study the electrostatic instability of the anode wire. The left view shows the optical system to observe the deflection of the anode wire. The right view shows the mechanical alignment of the tube.

Fig. 9. Illustration of the effect of an anode-wire offset as observed on the viewing screen.

Table 2. A sample of wire instability measurements.

T (gm)	D_t (mm)	V (V)	Measured displacement of anode wire D_w (in D_t unit)	V_0 (V)	V/V_0
10	0.045	779	0.5	1156	0.674
		918	1.0		0.794
		974	1.5		0.843
		1013	2.0		0.876
		1030	2.5		0.891
		1048	3.0		0.907
		1058	3.5		0.915
		1065	4.0		0.921
		1070	4.5		0.926
		1074	5.0		0.929
		1076	5.5		0.931
		1078	6.0		0.933
			unstable		
30	0.045	1349	0.5	2002	0.674
		1589	1.0		0.794
		1692	1.5		0.845
		1747	2.0		0.873
		1781	2.5		0.890
		1813	3.0		0.906
		1829	3.5		0.914
		1842	4.0		0.920
		1850	4.5		0.924
		1856	5.0		0.927
		1861	5.5		0.930
		1863	6.0		0.936
			unstable		

All data in Table 2 were taken with $R_a = 0.0127$ mm.

The electric force F_E between anode wire and cathode can be derived by differentiating W with respect to D:

$$F_E = \frac{2\pi\epsilon_0 V^2}{R_c^2(\ln(R_c/R_a))^2} D = KD = K(D_{\text{wire}} - D_{\text{tube}}),$$

where D denotes the total deviation of the anode wire and tube from perfectly symmetrical geometry; in more detail, D_{wire} is the deviation of the wire from

Fig. 10. Geometry of an anode wire offset by distance D from the center of the cathode cylinder of a straw-tube chamber.

Fig. 11. Coordinate system used in eq. (14)

straightness, and D_{tube} is the displacement of the center of the tube from the ideal straight-line of the anode wire.

On the other hand, the restoring force F_T at the midpoint of the anode wire of length L is

$$F_T = T\frac{d^2 D_{wire}}{dx^2},$$

where x is the coordinate along wire. For static equilibrium we have

$$F_T + F_E = 0,$$

$$T\frac{d^2 D_{wire}}{dx^2} + KD_{wire} = KD_{tube}. \tag{14}$$

The coordinate system used in eq. (14) is shown in Fig. 11. The solution of this equation depends on the knowledge of D_{tube}. Under the following two simple assumptions of D_{tube}: (1) $D_{tube} = D_t = const.$, (2) $D_{tube} = D_t \sin(\pi x/L)$, it is readily to solve the equation. The solutions are as following: with assumption (1):

$$D_{wire} = D_t \left\{ 1 - \frac{1-\cos(\alpha\pi)}{\sin(\alpha\pi)} \sin(\alpha\theta) - \cos(\alpha\theta) \right\}, \tag{15}$$

with assumption (2):

$$D_{wire} = D_t \frac{\alpha^2}{\alpha^2 - 1} \sin(\theta), \tag{16}$$

where $\alpha = \sqrt{K/T} L/\pi$, $\theta = \pi x/L$. Substitute $K = 2\pi\epsilon V^2/(R_c(\ln(R_c/R_a))^2)$ into α and notice $V_0 \equiv \sqrt{T/2\pi\epsilon}(\pi R_c^2/L)\ln(R_c/R_a)$, it follows $\alpha = V/V_0$. The general behavior of these solutions and our data points are shown in Fig. 12. In the figure $D_w = D_{wire}(x = L/2)$. The measurements are performed for five different cases: $R_a = 0.0102mm$, with $T = 10g, 20g, 30g$ and $R_a = 0.0127mm$, with $T = 10g, 30g$, respectively. The last point of each data set indicates that the anode wire starting vibration. By adjusting the position of the middle rod-end bearing we can set different D_t, which are indicated in Fig. 12. All of the data points are sitting in somewhere between theoretical curve $D_w/D_t = \alpha^2/(\alpha^2-1)\sin(\pi/2)$ and the half value of this theoretical curve $(1/2)(\alpha^2/(\alpha^2 - 1)\sin(\pi/2)$(curve (3) in Fig. 12), look like a comet tail. We attribute this discrepancy to the simplified theoretical assumption of $D_{\rm wire}$.

Given that a small lack of straightness, $D_{\rm tube}$, is inevitable we see that the anode wire will be deflected from its initial position as the voltage rises. After carefully adjusting the rod-end bearings 98% of V_0 had been reached. It confirms the correctness of eq. (13).

In Fig. 13 we plot V/V_0 *vs.* $D_{total} = D_w - D_t$ for those data points which had reached highest V before anode wire starting vibration (denoted as V_c). A linear line fits these data rather well. Since the starting points of vibration are pretty close to the theoretical curve(2) in Fig. 12, we may use eq. (16) to relate D_w/D_t and α for these data points. Combining the fitted line of Fig. 13 with eq. (16) at $\theta = \pi/2$, we finally get the function of D_t vs. V_c/V_0 as

$$D_t = \left(\frac{V_c}{V_0} + 1\right)\left(\frac{V_c}{V_0} - 1\right)^2 \Big/ 0.1248. \tag{17}$$

Fig. 14 shows this function as well as our data points. The data follows the curve very well. The general formula of D_t *vs.* $\alpha_c(\equiv V_c/V_0)$ appearently should include R_c, here we make a rather bold inference, using $1/R_c$ as a scaling factor to multiply both sides of eq. (17),

$$\frac{D_t}{R_c} = \left(\frac{V_c}{V_0} + 1\right)\left(\frac{V_c}{V_0} - 1\right)^2 \Big/ 0.3962. \tag{18}$$

The correctness of eq. (18) remains to be checked with different size of tubes, since it is derived from our data with only one size of the tube, the physical mechanism underlying the vibration setting in is still not clear.

Fig. 12(a). D_w/D_t *vs*. V/V_0 for $R_a = 0.0102mm$ case.

Fig. 12(b). D_w/D_t *vs*. V/V_0 for $R_a = 0.0127mm$ case.

Fig. 13. V_c/V_0 *vs*. D_{total} for five different cases.

Fig. 14. V_c/V_0 *vs.* D_t for five different cases.

Equation (16) also explains the experimental observations recently reported by Blockus *et al.*.[13]

Conclusions:

1. V_0 is an upper-limit high voltage for operating a straw tube chamber. 98% of this upper-limit value has been reached in our set-up.
2. For designing a straw-tube system, eq. (17), (18) and the curve (2), (3) in Fig. 12 may be used as the basic guideline to secure the stability of the system.
3. Any defect of roundness and straightness (including gravitational bending) of the straw tube itself will greatly affect the stability, as indicated by Fig. 14.

4. Effect of mechanical deviation on the gas gain

In a realistic straw-tube chamber there is always certain amount of mechanical deviation from the ideal symmetrical geometry. Therefore a gas-gain model, for example Diethorn's formula (1), should be modified as follows:

$$\ln G = \frac{V}{\cosh^{-1}(y)} \frac{\ln 2}{\Delta V} \left(\ln \frac{V}{P R_a \cosh^{-1}(y)} - \ln K \right),$$

where $y = (R_a^2 + R_c^2 - D^2)/2R_a R_c$, and D is the total displacement of the wire from the tube axis.

Fig. 15 shows the calculated curves of $\ln G$ *vs.* D for three different sizes of anode wire. It is evident that the thicker wire is more sensitive to mechanical deviation. In the case of $D = 0.5$ mm, the gas gain increases will be 5.6%, 8.3% and 11% for $R_a = 0.01$ mm, 0.02 mm and 0.03 mm, respectively.

5. Choice of the wire size — thick or thin?

Fig. 15. Calculated effect of a displacement D of the anode wire on the gas gain.

Table 3. Pros and cons of thick and thin wires.

Effect	Thick wire	Thin wire
Gravitational sagitta	same	same
Effect of mechanical deviation on gain	worse	better
Temperature dependence	worse	better
Electrostatic stability	better	worse

The choice of the wire size is a compromise among various considerations. We summarize some of them in Table 3.

Some supporting remarks:

Gravitational sagitta – For a tungsten wire of L (cm) in length, R_a (cm) in radius, under tension T (gm) and mounted horizontally, the sagitta is

$$s = 7.58 L^2 R_a^2 / T,$$

while the maximum practical tension varies at $T \propto R_a^2$ due the the breaking strength of the wire.

Mechanical deviation – see Fig. 10.

Temperature dependence – see Figs. 5, 6 and 7.

Electrostatic stability – Eq. (13) indicates that $V_0 \propto \sqrt{T}\ln(R_C/R_A)$, while $T \propto R_a^2$, so $V_0 \propto R_a \ln(R_c/R_a)$. On the other hand, the dependence of the gas gain on the wire radius is much slower than this, as shown by Diethorn's formula (1).

6. An example

Since an Ar/Ethane (50/50) mixture has been used in the AMY vertex and inner-tracking chambers, and also was tested by MAC vertex-detector group, it is one of the best-understood gas mixtures for straw-tube chambers. We use the gas parameters of this mixture as found above to make a sample design of a straw-tube system.

We set the geometry of straw-tube chamber as

$$R_a = 0.00102 \text{ cm}, \qquad R_c = 0.35 \text{ cm}, \qquad L = 200 \text{ cm}.$$

In this example, the straw is mounted horizontally.

We use Aoyama's formula (2) for gas-gain calculations, with the parameters $A = 0.1141 \times 10^{-6}$, $m = 0.4942$, $V_I = 12.86$ V; see Fig. 3.

If we want to operate our chamber at $G \approx 2.5 \times 10^4$, the high voltage should be set at $V = 1550$ V.

From Fig. 7(c) we find $(dG/G)/(dT/T) \approx 4.9$ under the present circumstances.

If we set tension $T = 40$ gm, the gravitational sagitta will be 75 μm, and the critical high voltage is $V_0 = 2696$ V, yielding $V/V_0 = 57.5\%$.

According to eq. (16) the wire displacement D_w caused by electric field will be

$$D_w = \left[\left(\frac{V_0}{V}\right)^2 - 1\right]^{-1} \times 75\ \mu\text{m} = 37\ \mu\text{m}.$$

Therefore the total displacement of the anode wire from the tube center is

$$D_{total} = 75 + 37 = 112\ \mu\text{m}.$$

This displacement will cause negligible variation of the gas gain, but will require a large correction to the position measurement.

Now we switch to a fat wire, $R_a = 0.0254$ mm. If we still desire $G = 2.5 \times 10^4$, the high voltage should be set at $V \approx 1960$ V.

A tension of 40 gm for $R_a = 0.0102$-mm wire scales up to $T = 250$ gm at $R_a = 0.0254$ mm. It follows that

$$V_0 = 5684 \text{ V},$$

Table 6. Summary of the design exercise.

R_a (mm)	0.0102	0.0254	0.0102	0.0254
Gas pressure (atm)	1	1	3	3
Tension T (gm)	40	250	40	250
Gain G ($\times 10^4$)	2.5	2.5	2.5	2.5
V (V)	1550	1960	2550	3450
V_0 (V)	2696	5684	2696	5684
V/V_0 (%)	58	34	95	60
D_w^{grav} (μm)	75	75	75	75
D_w^E (μm)	37	10	694	42
D^{Total} (μm)	112	85	769	117
$(dG/G)/(dT/T)$	4.9	6.6	6.8	9.2

$$V/V_0 = 34.4\%,$$

$$D_w = \left[\left(\frac{V_0}{V} \right)^2 - 1 \right]^{-1} \times 75\ \mu\text{m} = 10\ \mu\text{m}.$$

Using Aoyama's formula (2) we can predict

$$\frac{dG/G}{dT/T} \approx 0.7 \times 9.5 = 6.6.$$

The spatial resolution attainable with Ar/Ethane (50/50) at 1 atm. is only about 120 μm according to the measurements of MAC group[14] and AMY group[8,15]. In order to improve the spatial resolution high gas pressure could be used. If we run our chamber at 3 atm, the calculated results are summarized in Table 6. Note that a thin anode wire is not indicated at high pressure because the higher voltage required there renders the tube susceptible to the electrostatic instability.

7. Conclusion

1. The gas-gain models of Diethorn, Aoyama, and of Kowalski fit our experimental data (at a fixed temperature) rather well.

2. Experimental measurements have been made of the temperature dependence of the gas gain. The gas-gain formulae indicate the general behaviour of this dependence, but there is a substantial numerical disparity between data and the models, and the model values should be used only as an upper-limit estimate of temperature effects.

3. The voltage V_0 of eq. (13) is a theoretical upper-limit value for stable operation of a straw-tube chamber: 98% of this value could be reached, for designing a large straw-tube system, eq. (16),(17) should be consulted to assure the stable operation of the chamber and the straightness of the anode wire itself.

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