The Guard Ring of a Streamer Chamber

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1 Problem

The charge density and electric fields near the edge of a conducting sheet vary as $1/\sqrt{s}$, where distance s is measured in from the edge. Thus, very high fields occur near the edge, and physical devices such as parallel plate capacitors are subject to electrical breakdown due to field emission at such edges.

A so-called streamer chamber is a device in which a pulse of high voltage is applied to a box of gas contained within a parallel plate capacitor. If a high energy particle passed through the box just before the pulse, the ionized atoms along the particle's path will be pulled apart by the pulse, each atom creating a region of further ionization. The light from the subsequent recombination of the secondary ionizations appears as a "streamer", which can be photographed to visualize the path of the high energy particle.

To avoid breakdown of the high voltage electrode, a conducting "guard ring" of radius a is attached around the edge of the electrode. The high voltage electrode lies between a pair of ground electrodes to form a double gap capacitor, as shown in Fig. 1, and extends distance $b \gg a$ beyond the ground electrodes. Each gap of the chamber has height $h \ll b$.

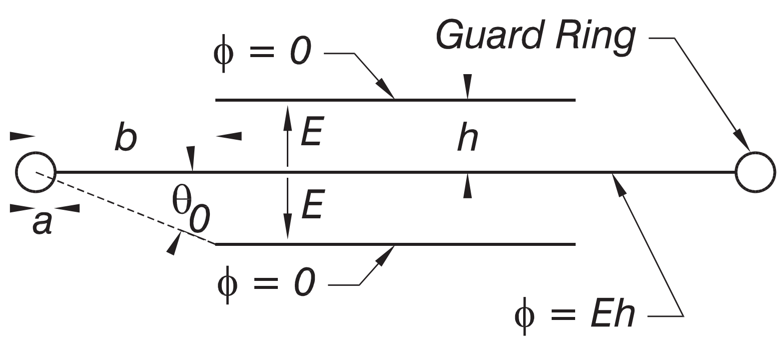

Figure 1: A double gap streamer chamber of gap height h. The high voltage electrod extends distance b beyond the ground electrodes, and is terminated in a conducting guard ring of radius a.

If the central electric field in each gap is E, deduce the value of the maximum electric field on the guard ring.

This problem may be analyzed by considering the two-dimensional region of radius b centered on the guard ring. The boundary condition at $r = b$ is approximately

$$\phi(r = b) \simeq \begin{cases} Eh(1 - \theta/\theta_0), & |\theta| < \theta_0, \\ 0, & \theta_0 < |\theta|\pi, \end{cases} \tag{1}$$

where $\theta_0 \approx h/b \ll 1.$ and h is the gap height.

2 Solution

The approach is to consider the electrostatic problem of the two dimensional region $a < r < b$, $0 \le \theta \le \alpha$, where $\alpha \to 2\pi$, in cylindrical coordinates (r, θ), as shown in Fig. 2. This region is bounded by conducting surfaces held at potential Eh, except for the surface at $r = b$ where condition (2) holds.

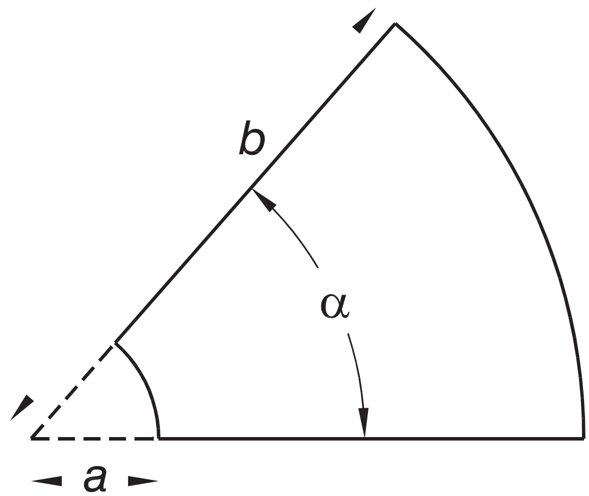

Figure 2: The two-dimensional region $a < r < b$, $0 < \theta < \alpha$ used to analyze the streamer chamber guard ring.

Since we are interested in the electric fields, it is equivalent to consider the slightly simpler problem obtained by subtracting potential $\phi = Eh$ everywhere. Now the boundary conditions that $\phi = 0$ for $r = a$ and for $\theta = 0$ and α, and

$$\phi(r = b) \simeq \begin{cases} -Eh\theta/\theta_0), & |\theta| < \theta_0, \\ -Eh, & \theta_0 < |\theta|\pi, \end{cases} \tag{2}$$

for $r = b$.

Laplace's equation, $\nabla^2\phi = 0$, holds for the potential in the interior of the region. The general form a series expansion for ϕ in such a region is

$$\phi(r, \theta) = \sum_{n=1}^{\infty} \left[a_n \left(\frac{r}{a}\right)^{k_n} + b_n \left(\frac{a}{r}\right)^{k_n} \right] \left[c_n \cos k_n\theta + d_n \sin k_n\theta \right]. \tag{3}$$

Since $\phi(r, 0) = 0 = \phi(r, \alpha)$, the angular factors can only be $\sin(n\pi\theta/\alpha) = \sin(n\theta/2)$ for $\alpha = 2\pi$. Since the radial extent includes neither the orgin nor ∞, factors of both $r^{n/2}$ and $r^{-n/2}$ can occur. Thus, the potential can be written

$$\phi(r, \theta) = \sum_{n=1}^{\infty} \left[a_n \left(\frac{r}{a}\right)^{n/2} + b_n \left(\frac{a}{r}\right)^{n/2} \right] \sin \frac{n\theta}{2}. \tag{4}$$

The use of factors r/a and a/r is convenient for enforcing the boundary condition $\phi(a, \theta) = 0$, since this simply requires $b_n = -a_n$.

For the electric field, we get:

$$E_r = -\frac{\partial \phi}{\partial r} = -\sum_n \frac{n\pi}{\alpha} a_n \left[\frac{1}{r}\left(\frac{r}{a}\right)^{n/2} + \frac{1}{r}\left(\frac{a}{r}\right)^{n/2} \right] \sin\frac{n\theta}{2}, \qquad (5)$$

$$E_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\sum_n \frac{n}{2} a_n \left[\frac{1}{r}\left(\frac{r}{a}\right)^{n/2} - \frac{1}{r}\left(\frac{a}{r}\right)^{n/2} \right] \cos\frac{n\theta}{2} \qquad (6)$$

At $r = a$, $E_\theta = 0$. At $\theta = 0$, $E_r = 0$, as expected. At $r = a$,

$$E_r = -\frac{1}{a}\sum_n n a_n \sin\frac{n\theta}{2}. \qquad (7)$$

At $\theta = 0$,

$$E_\theta = -\frac{1}{2r}\sum_n n\, a_n \left[\left(\frac{r}{a}\right)^{n/2} - \left(\frac{a}{r}\right)^{n/2} \right]. \qquad (8)$$

As expected, E_θ vanishes on the conducting surface $r = a$.

We use the boundary condition (2) at $r = b$ to determine the coefficients a_n. Inserting (4) in (2), multiplying by $\sin(n\theta/2)$ and integrating from 0 to π, we find

$$n\, a_n = \frac{2Eh}{\pi}\left(\frac{a}{b}\right)^{n/2}\left(\cos\frac{n\pi}{2} - \frac{2}{n\theta_0}\sin\frac{n\theta_0}{2}\right). \qquad (9)$$

To obtain this, we ignored the small terms proportional to $(a/b)^{n/2}$ in $\phi(r = b)$ compared to those proportional to $(b/a)^{n/2}$.

From (7), the field on the surface of the guard cylinder is

$$E_r(r = a) = -\frac{2Eh}{\pi a}\sum_n \left(\frac{a}{b}\right)^{n/2}\left(\cos\frac{n\pi}{2} - \frac{2}{n\theta_0}\sin\frac{n\theta_0}{2}\right)\sin\frac{n\theta}{2}\,. \qquad (10)$$

Since a/b is very small, it suffices to keep only the first term, which is maximal at $\theta = \pi$:

$$E_{r,\max}(r = a) \simeq \frac{2Eh}{\pi\sqrt{ab}}. \qquad (11)$$

As desired, the field on the guard ring satisfies $E_{r,\max} < E$ for reasonable values of a, b and h.

A sign that our approximations are somewhat delicate is obtained by evaluating $E_\theta(r = b)$ using (8). If we keep only the first term, we find

$$E_\theta(r = b) \approx \frac{Eh}{\pi b}, \qquad (12)$$

instead of E. However, because of the form of the a_n, the terms in series (8) do not have any factors of a/b, and this series converges much more slowly than does (7). The terms are of similar magnitude until $n\theta_0 \approx \pi$, *i.e.*, until $n \approx \pi b/h$, and $E_\theta(r = b)$ sums to E.