

# Ph 406: Elementary Particle Physics

## Problem Set 1

K.T. McDonald  
kirkmcd@princeton.edu  
*Princeton University*

Due Monday, September 22, 2014 (updated August 19, 2016)

The lecture notes for this course are more or less those at <http://physics.princeton.edu/~mcdonald/examples/index.html#ph529>  
The problem sets will be posted at <http://physics.princeton.edu/~mcdonald/examples/index.html#ph406>  
Many papers related to this course are in my password-protected directories with links at the top of <http://physics.princeton.edu/~mcdonald/examples/>  
Example: A lecture by Yang at Princeton in 1960 about particle physics just prior to the quark model: [http://physics.princeton.edu/~mcdonald/examples/EP/yang\\_elementary\\_particles.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/yang_elementary_particles.pdf)

1. In this course we will often use “natural” units in which  $\hbar = c = 1$ , where  $\hbar = h/2\pi$  is Planck’s constant and  $c$  is the speed of light in vacuum. If we also define the unit of mass/energy to be the mass of the proton, what are the units of length and time in this system, expressed in SI units.

More commonly, we will take the unit of energy to be 1 GeV (or 1 MeV) in our “natural” units. Since the mass of the proton is 0.94 GeV (in these units), the units of length and time in the GeV- $\hbar$ - $c$  system differ only by 6% from the values you computed above. That is, a GeV is a rather “natural” unit of energy in a system that emphasizes protons.

2. The cross section  $\sigma$  for the reaction  $\pi^- + p \rightarrow \Lambda^0 + K^0$  is about 1 mb =  $10^{-27}$  cm<sup>2</sup>. Noting that  $m_\pi \approx m_p/6$ , estimate the dimensionless coupling constant  $\alpha_S$  for the (strong) interaction of this process. *Hint:  $\sigma \approx \alpha_S^2 \sigma_{\text{geometric}}$  in the lowest approximation, which suffices for this problem.*

The lifetimes  $\tau$  of the  $\Lambda^0$  and  $K_0$  particles are both around  $10^{-10}$  s. Noting the  $m_\Lambda \approx 2m_K \approx m_p$ , estimate the dimensionless coupling constant  $\alpha_W$  that is relevant to the (weak) decay processes for these particles.

What is your estimate for the ratio  $\alpha_W/\alpha_S$  of the relative strengths of the weak and strong interactions?

3. Use classical electrodynamics to deduce the Thomson scattering cross section  $\sigma_{\gamma e \rightarrow \gamma e}$  for the scattering of unpolarized light by an electron nominally at rest. *Hint: Rather than slogging through a derivation based on the differential cross section, as in the text of Jackson, note that  $\sigma = P_{\text{scattered}}/S_{\text{incident}}$ , where  $\mathbf{S}$  is the Poynting vector and  $P_{\text{scattered}} = P_{\text{radiated}}$  where the latter can be gotten quickly from the so-called Larmor formula. And, it’s simpler to use Gaussian units if you are familiar with these.*

4. The so-called quantum electrodynamic critical field strength  $E_{\text{crit}}$  is that such if an electron were accelerated in this (static, uniform) field for a distance equal to the (reduced) Compton wavelength  $\lambda_C$  of an electron, it would gain energy equal to its rest mass. Deduce an expression for  $E_{\text{crit}}$  (in Gaussian units), but give a numerical value for it in the hybrid units of volts/cm.

One also speaks of the critical magnetic field strength,  $B_{\text{crit}} = E_{\text{crit}}$ . Deduce the value of  $B_{\text{crit}}$  in gauss, which is the field strength at the magnetic poles of some neutron stars (called **magnetars**).

If the QED critical field strength could be achieved, the “vacuum” would “spark,” in that “virtual” electron-positron pairs of nominally zero mass would be given enough energy by such a field, while still separated by the size  $\lambda_C$  of the quantum fluctuation for the particles to become “real” with mass/energy  $mc^2$ .

What is the electric field strength at the surface of a lead nucleus, in units of  $E_{\text{crit}}$ ?

Note that if a “virtual” electron-positron pair is created (with zero rest energy) out of the vacuum near a nucleus, the electron could be captured into an atomic level with binding energy  $U$ , and this energy given to rest energies of the electron and positron. If  $U > 2m_e c^2$ , then the electron and positron become “real,” and we say that the vacuum has “sparked;” otherwise the electron-positron pair must go back into the “vacuum.” Use a nonrelativistic Bohr model of an atom with a nucleus of charge  $Ze$  to predict the minimum value of  $Z$  such that this kind of “sparking the vacuum” could occur. *Relativistic corrections reduce this  $Z_{\text{crit}}$  significantly. Hint: express the parameter of an atom in terms of  $\lambda_C$ , the electromagnetic coupling constant  $\alpha_{EM} = \alpha = e^2/\hbar c$ , and the electron rest energy  $m_e c^2$ .*

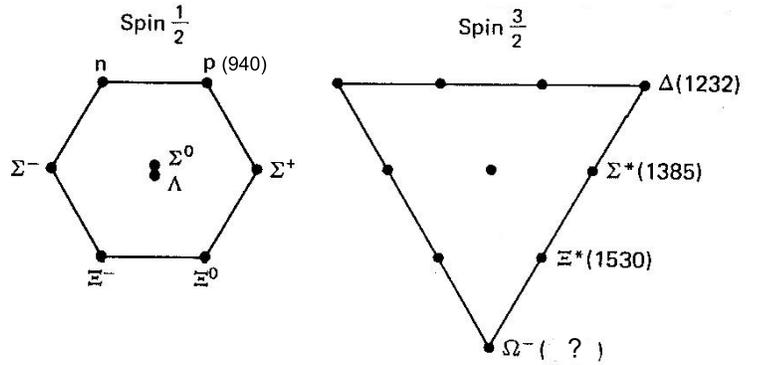
*Searches for “sparking the vacuum” in collisions of uranium nuclei, where briefly the total  $Z$  is 184, have led to ambiguous results. In an experiment by the author, electron-pairs were produced when a high-energy photon probed a very intense laser beam, whose electric field strength was close  $E_{\text{crit}}$  in the rest frame electron-positron pair; the results can be interpreted in the complementary ways of “sparking the vacuum” or the nonlinear reaction  $\gamma + n\gamma_{\text{laser}} \rightarrow e^+e^-$ . See sec. IVb of C. Bamber *et al.*, *Phys. Rev. D* **60**, 092004 (1999),*

[http://physics.princeton.edu/~mcdonald/examples/QED/bamber\\_prd\\_60\\_092004\\_99.pdf](http://physics.princeton.edu/~mcdonald/examples/QED/bamber_prd_60_092004_99.pdf)

*Note that a strong laser beam (plane electromagnetic wave) cannot by itself “spark the vacuum” in that an electron-positron pair has a rest frame, while there is no rest frame for a collection of identical photons.*

5. Using the information given in the diagram of the baryon octet and decuplet, with masses in MeV, predict the (constituent) masses of the  $u$ ,  $d$  and  $s$  quarks, and the mass of the  $\Omega^-$  baryon.

*The latter prediction was the only one from the quark model that was verified between its development and its Nobel Prize.*



6. What would the characteristic binding energy and radius of a possible neutron-electron atom (bound state) as a result of the force between their magnetic moments?

You could recall certain general theorems of classical mechanics, and/or give a semi-classical argument in the spirit of Bohr, supposing that the two magnetic moments are (anti)parallel. The magnitudes of the magnetic moments can be written as

$$\mu_e = g_e \frac{e\hbar}{2m_e c} = \frac{g_e}{2} e\lambda_e, \quad \mu_n = g_n \frac{e\hbar}{2m_n c} = \frac{g_n}{2} \frac{m_e}{m_n} e\lambda_e, \quad (1)$$

where  $g_e \approx 2$ ,  $g_n \approx 2.8$ ,  $e$  is the magnitude of the charge of the electron,  $m_e$  and  $m_n$  are the masses of the electron and neutron,  $c$  is the speed of light, and  $\lambda_e = \hbar/m_e c \approx 3.9 \times 10^{-11}$  cm.