

7 Physics in Collision

Introduction

When two objects collide it is typically impossible to follow the details of the process because its duration is too short. Rather, we can measure the ‘before’ and ‘after’ conditions and then try to infer what happened ‘during’.

For collisions in which no outside forces act Newton’s third law guarantees that the sums of the internal forces and torques are both zero. This implies that momentum and angular momentum are conserved during the collision. Kinetic energy may or may not be conserved during collisions.

In this laboratory you will minimize the horizontal forces on colliding objects (called pucks) by means of an air table (as popularized in air hockey games). You will examine the validity of the three basic conservation laws in collisions involving three types of pucks:

1. Plain pucks that bounce well off each other.
2. Magnetic pucks that can collide without physical contact.
3. Velcro pucks that stick to one another after colliding.

Observations of the collisions will be made with a CCD camera similar to that in home TV cameras. The CCD camera is interfaced to a computer and controlled by an MS-Windows-based program. Once images are captured the computer can assist you in their analysis. The variety and power of the computer analysis tools is such that there is not a unique best way to perform the data analysis. This writeup suggests suitable methods for the analysis, but if you wish to explore your own variation you are encouraged to do so.

7.1 Collisions of Plain Pucks

The collisions of plain pucks will be analyzed to verify conservation of momentum and to determine to what extent energy is conserved.

7.1.1 Preparations to Capture Images with the CCD Camera

Camera Setup

If the computer is in dos type win to enter Windows. Select Physics 101 and then Camera to start the camera program.

Once you are in the EDC-1000 Imaging Software window, click on the box to the right of Live Mode until it is On. The image window should now show real-time images, although they may be dark. If the brightness of the image needs adjustment you can do this without climbing on the table by means of a 2-meter stick. Gently move the small arm projecting from the camera with the 2-meter stick until the desired exposure level is achieved.

To take a picture, click on the box to the right of Single Frame. If you’d like to save the photo, click on File and then on Save Image. Use a filename with a .tif extension.

Pull down the Image Capture menu, and click on Set Exposure Time. A suitable value is between 10 and 15 milliseconds (the default of 200 ms is too long for this lab). The

Delay Time should be set to zero. The Scan Time is not adjustable; there will always be 53 milliseconds of computer processing time between successive images during multiframe scans. Also, Set MFA Frame Number to 16, which will permit a multiple exposure of 16 images of the collision process.

Puck Setup

The analysis of collisions is simpler if there is no other image captured than the collision itself. To this end you will 'stop down' the lens of the camera until it appears dark. The pucks will be visible only because of an LED attached to their center.

If LED's are not already mounted on the plain pucks, unscrew the LED's and housing from the other pucks and remount them on the plain pucks. The LED is turned on by inserting the U-shaped clip in the pair of holes on the housing. As the battery life is short, PLEASE REMOVE THE CLIPS TO TURN OFF THE LED'S WHEN YOU ARE NOT TAKING DATA. The clip can be stored in the housing by inserting only one leg into a hole.

If necessary, readjust the f -stop of the CCD camera until only the LED's from the pucks are visible. You may find that a vertical stripe appears associated with the LED. This is called streaking or 'blooming' and is a limitation of CCD technology in the presence of localized bright spots. Some amount of streaking is acceptable, and can be compensated for later in the computer analysis.

Once the exposure level of the camera has been set you should capture a 'dark frame'. For this, first remove all pucks, hands, etc., from the air table and click on the box to the right of Background Frame. The dark frame can later be subtracted from your images of collisions to suppress any constant background such as the edge of the table.

Trial Images of Collisions

One member of your group will launch the pair of pucks on a collision course, and another will control the picture taking with the computer. Launch the pucks from adjacent corners of the air table, aiming them to produce a collision near the center of the table.

Click on the box labelled MFA Start to bring up the Multiple Frame Acquisition window. Just after the pucks are launched, press the spacebar to start the multiple exposure.

The computer takes a while to read out the camera and then displays the image. However, this image will likely appear entirely black! Before you can see the trajectories of the pucks you must first do some image processing. Click on Image Utilities and then on Subtract Background Image, and then on Optimized Contrast. Typically only after the second processing step will the trajectories appear.

Go through the capture and processing cycle a few times. A good collision image should have about 8 points before the collision and 8 points after for each trajectory.

If you are satisfied with your image, switch it to Negative using the Image Capture menu. Then Save Image with a .tif extension, and Print. Please don't Print an image with a dark background; use Negative (or Print Negative) to convert the image to one with a light background.

7.1.2 Data Analysis

Each group should collect one good image of a collision between the plain pucks. An example of a collision between two plain pucks is shown in Fig. 1.

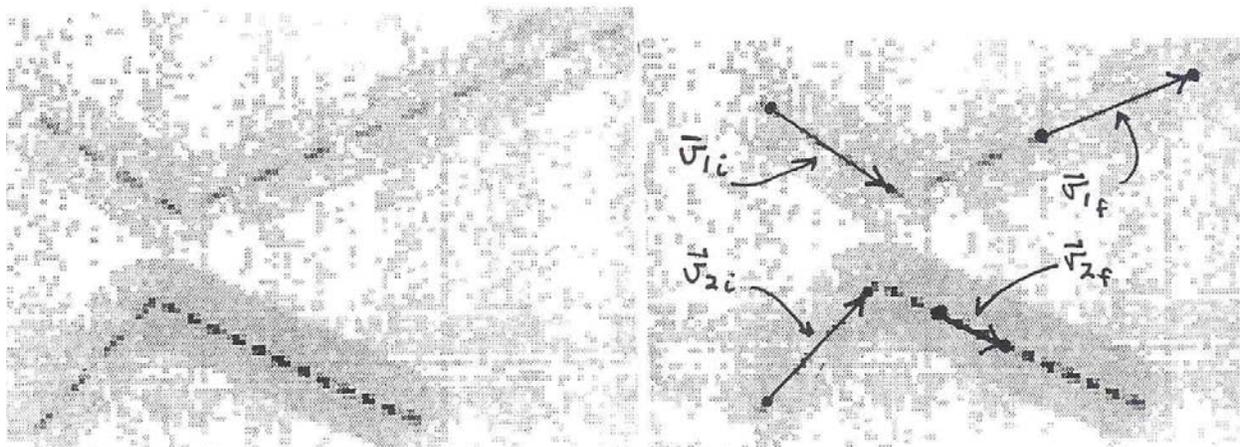


Figure 1: A collision between plain pucks. A possible definition of the heads and tails of the velocity vectors is shown on the right. It is advantageous to define each vector as spanning the same number of time intervals.

We wish to reconstruct the initial and final velocities of the pucks, and then examine the validity of the conservation laws

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (\text{momentum}) \quad (1)$$

and

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (\text{energy}). \quad (2)$$

The masses of the two pucks are equal. Furthermore, in this Lab we will be concerned only with relative values, so it is convenient to set masses m_1 and m_2 to 1. Thus we want to examine the validity of the simplified relations

$$\vec{v}_{1i} + \vec{v}_{2i} = \vec{v}_{1f} + \vec{v}_{2f} \quad (\text{momentum}) \quad (3)$$

and

$$v_{1i}^2 + v_{2i}^2 = v_{1f}^2 + v_{2f}^2 \quad (\text{energy}). \quad (4)$$

Since the time interval is constant between the successive multiple exposures the distance between two adjacent points on the trajectory of a puck is proportional to the velocity. So the task of measuring the velocities reduces to that of measuring distances. (We declare the time interval between points to be one Ph101 second!) You can use the computer to measure the relevant distances.

Before minimizing the EDC-1000 Imaging Software window to do your analysis, click on the box next to Live Mode until it is Off to free up the computer for other tasks.

Click on the Point-n-Click icon of the Physics 101 window to bring up the first analysis window. Click on File and then Open Image, and activate your image file from the directory listing.

Click on Point/Vector Mode and then on Vector. Position the mouse on the first point on the trajectory of one puck and click. Position on the farthest point still on the initial straight-line segment of the trajectory and click again. The magnitude and direction of the vector you have just defined will appear in the data box.

Digitize a vector for the initial and final movement of each of the two pucks. If you need to erase a data point, first highlight it within the data box using the mouse, then click on Clear Point. When you have a complete set of digitized vectors, save it to disk via Save Data of the File menu. It is helpful to use a .txt extension.

If the different digitized vectors span different time intervals you must divide each vector length by the time interval to obtain the velocities. But if you have digitized each vector with the same number of time intervals there is no need for this. For example, in Fig. 1 each vector is conveniently digitized over three time intervals.

To print the table of your vectors you must leave Point-n-Click, preferably by minimizing the window. You can then either use the Notepad or StatMost. Your data file is in the directory c:\phys101. It is a little more work to import the data into StatMost, but that program can be used to do part of the data analysis.

7.1.3 Conservation of Momentum

To verify momentum conservation, examine the x and y components of the momentum:

$$\begin{aligned} p_{xi} &= v_{1i} \cos \theta_{1i} + v_{2i} \cos \theta_{2i}, \\ p_{yi} &= v_{1i} \sin \theta_{1i} + v_{2i} \sin \theta_{2i}, \\ p_{xf} &= v_{1f} \cos \theta_{1f} + v_{2f} \cos \theta_{2f}, \\ p_{yf} &= v_{1f} \sin \theta_{1f} + v_{2f} \sin \theta_{2f}. \end{aligned} \quad (5)$$

Report the relative accuracy of momentum conservation as

$$\frac{\Delta p}{p} = \frac{\sqrt{(p_{xi} - p_{xf})^2 + (p_{yi} - p_{yf})^2}}{\sqrt{p_{xi}^2 + p_{yi}^2}}. \quad (6)$$

StatMost can readily do the first steps in the analyses of secs. 7.1.3 and 7.1.4. Open a New Sheet Document and Import your data file Delimited with Commas. The vector lengths (velocities) will be in column A and the angles (in degrees) in column B. You could use Data, then Transform, then Simple Math with Formulas $VX=A*\text{COS}(B/57.296)$, $VY=A*\text{SIN}(B/57.296)$, and $VSQ=A^2$ to calculate v_x , v_y and v^2 , respectively. Further manipulations become increasingly cumbersome compared to the simplicity of a hand calculation. Perhaps a speed test is in order between part of the group on the computer and the others using calculators.

7.1.4 Conservation of Energy

Energy conservation can also be examined by calculating

$$\frac{E_f}{E_i} = \frac{v_{1f}^2 + v_{2f}^2}{v_{1i}^2 + v_{2i}^2}. \quad (7)$$

It is likely that a substantial fraction of the initial kinetic energy was lost in the collision. In part 2 you will examine a situation in which kinetic energy may be better conserved.

7.2 Collisions of Magnetic Pucks

[Analyze part 1 before proceeding.]

It is very possible that a significant amount of kinetic energy was lost in part 1 during the impact of the two pucks. Perhaps energy would be better conserved if the pucks could interact without physically touching one another. This can be arranged via the use of magnetic pucks. The ‘fields’ of small magnets inside these pucks extend beyond the physical boundaries of the pucks and will cause them to deflect one another even without direct contact.

Transfer the LED housings from the plain pucks to the magnetic pucks. To observe the magnetic deflection of the pucks their velocities must be low. To prevent the multiple images from overlapping, click on Set Delay Time of the Camera Parameters menu and choose 100 milliseconds as the additional time between exposures. For this you will first need to reactivate the EDC-1000 Imaging Software window.

Practice launching the magnetic pucks so they deflect but do not touch. Adjust the Delay Time if the spacing between the multiple images is consistently too small or too large. Save a good image to disk after image processing as on p. 2. Print a copy of your image, again using the Negative option.

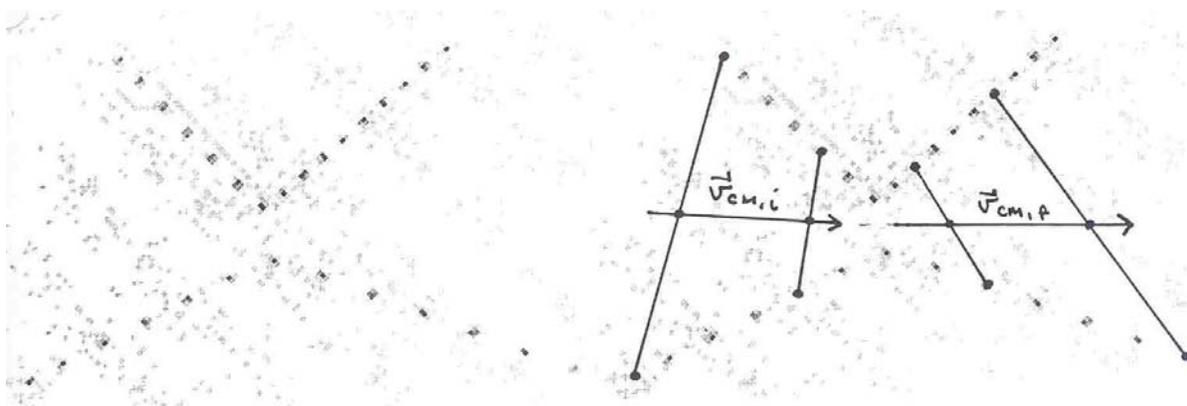


Figure 2: A collision between magnetic pucks. The graphical analysis of the center-of-mass motion is shown on the right.

An example of a collision between magnetic pucks is shown in Fig. 2. The image looks similar to that of a collision between plain pucks. However, close examination shows that

the centers of the pucks during the three frames closest to the collision point do not lie on straight lines extrapolated from earlier and later times.

7.2.1 Conservation of Momentum

While momentum conservation could be verified by a repeat of the analysis of part 1, try the following alternative. Momentum conservation implies that the center of mass moves with the same velocity (both magnitude and direction) before and after the collision. Since the magnetic pucks have equal masses the center of mass of the two puck system lies halfway between the centers of the two pucks. Use a ruler to draw a line connecting the two pucks for each of the 16 frames on the printout of your image. Draw a dot at the midpoint of each of these lines. How well does a single straight line fit through all 16 center of mass points? Is the distance between neighboring points uniform?

7.2.2 Conservation of Energy

To analyze energy conservation you could digitize velocity vectors as in part 1. Or you could just measure the distance of the vectors with a ruler on your printout of the collision. Either way, only use points on the initial and final straight-line segments of the pucks trajectories, avoiding points close to the collision; the trajectories of the magnetic pucks are curved when the pucks are close together. Report E_f/E_i as in eq. (7). If this value is significantly less than 1, comment why this might be so.

7.3 Collisions of Velcro Pucks

[Analyze part 2 before proceeding.]

In the last part of this Lab you will explore angular momentum conservation in collisions.

Transfer the LED housing from the magnetic pucks to the pucks with the Velcro bands. When the Velcro pucks collide they stick together and typically spin about their overall center of mass. Practice launching the pucks so they do not spin before the collision. Since the camera does not observe the initial spin you could not include it in your analysis of angular momentum.

After a ‘good’ collision the two-puck system rotates about 180° about its center of mass before hitting the edge of the air table. You may want to restore the Delay Time to zero, since higher puck velocities can be used in this part. When you have a ‘good’ image Save it and Print it. An example of a collision between Velcro pucks is shown in Fig. 3.

To begin your analysis, locate the center of mass of the two pucks for each of the 16 frames on the printout of your image. Again, a single straight line should fit through all of the center-of-mass points if momentum is conserved.

Note that the lines joining the centers of the two pucks in each frame are rotated from frame to frame. This rotation is readily visible to you after the collision when the pucks are stuck together. But your graphical analysis reveals that a rotation was present before the collision as well. You will now analyze that rotation in more detail.

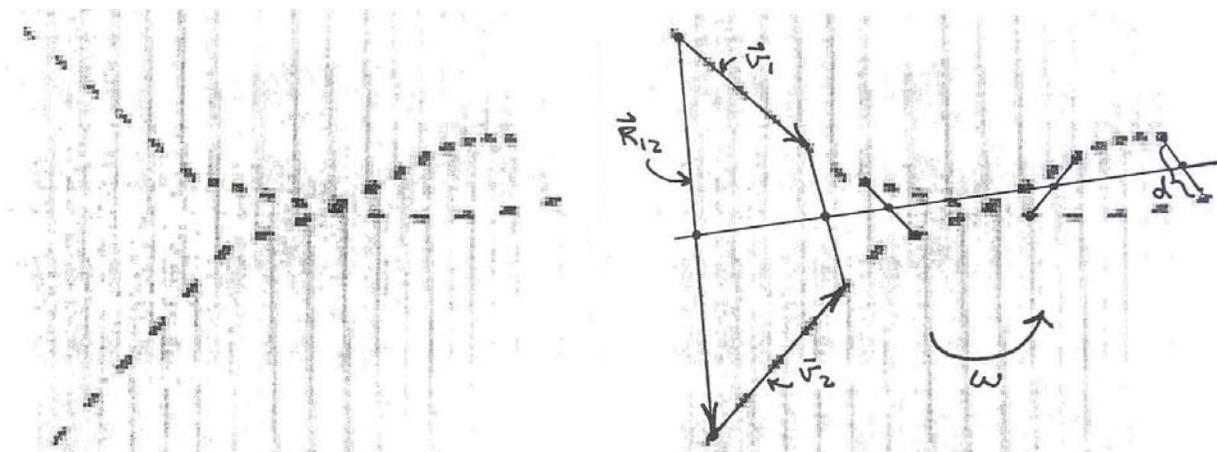


Figure 3: A collision between Velcro pucks. Part of the graphical analysis of the initial and final angular momentum is shown on the right.

7.3.1 Conservation of Angular Momentum

In this part you will quantify the angular momentum of the two-puck system.

Start with the situation after the collision, for which the pucks rotate about their common center of mass with angular velocity ω while the center of mass moves with constant velocity \vec{v}_{cm} . The final angular momentum about the center of mass is

$$L_f = I\omega. \quad (8)$$

where I is the moment of inertia of the two-puck system.

The moment of inertia can be calculated as follows. Each puck has moment of inertia $I_0 \approx \frac{1}{2}mr^2$ about its own center of mass, where r is the radius of the puck. But after the pucks have stuck together they rotate about their common center of mass, which is distance r from the center of each puck. According to the ‘parallel-axis theorem’ (eq. (45) of Lab 5) the moment of inertia of a puck about a point a distance r from its center is $mr^2 + I_0 = \frac{3}{2}mr^2$. The total moment of inertia of the two pucks is thus $3mr^2$. As before we define the pucks to have mass $m = 1$ so that

$$I = 3r^2 = \frac{3}{4}d^2, \quad (9)$$

where d is the distance between the centers of the two pucks when they are stuck together.

The angular velocity ω can be measured as follows. [An alternative graphical analysis is suggested in part 7.3.3 if you prefer not to use the computer.] *Open the Point-n-Click window and open your image file of the Velcro puck collision. Use the Vector mode to digitize the line joining the centers of the two pucks just after they have stuck together. For example, if the collision occurred between the 6th and 7th frames, connect the 7th point on each puck’s trajectory to form this vector. Digitize another vector using the last pair of points on each puck’s trajectory. Be sure that the first point on each of these vectors lies on the trajectory of the same puck. Let n be the number of time intervals between the two vectors. Then if*

the first frame after the collision is the i th frame the last frame is the $i + n$ th, and

$$\omega = \frac{\theta_{i+n} - \theta_i}{57.3n}. \quad (10)$$

Because the Point-n-Click program reports the angles in degrees, you must divide by 57.3° to convert the angular velocity to radians/sec.

If the combined pucks were spinning extremely rapidly they might have rotated by more than a full circle. If so, you will need to increase (or decrease) $\theta_{i+n} - \theta_i$ by 360° in eq. (10).

The distance d between the centers of the two pucks is just the length of the vector you have digitized. With this you can calculate I and L_f using eqs. (8-10).

Now you will calculate the angular momentum of the initial state.

It may appear that there is no angular momentum in the initial state, since you carefully launched the pucks so that they didn't rotate. However, recall eq. (28) of Lab 5 that the basic definition of angular momentum is

$$\vec{L} = \vec{r} \times m\vec{v}. \quad (11)$$

In the present case the angular momentum is measured with respect to the center of mass of the two-puck system. Setting the masses of the pucks to 1 as usual, the initial angular momentum can be written

$$\vec{L}_i = \vec{r}_1 \times \vec{v}_1 + \vec{r}_2 \times \vec{v}_2, \quad (12)$$

where vectors \vec{r}_1 and \vec{r}_2 extend from the center of mass to pucks 1 and 2. [This expression is correct only if the pucks were NOT spinning initially.]

Since the pucks have equal mass their common center of mass lies exactly halfway between them, which implies that $\vec{r}_1 = -\vec{r}_2$. Furthermore, vector \vec{r}_2 is exactly one half of vector \vec{R}_{12} which extends from the center of puck 1 to that of puck 2. Hence

$$\vec{L}_i = \frac{\vec{R}_{12}}{2} \times (\vec{v}_2 - \vec{v}_1). \quad (13)$$

The magnitude of the initial angular momentum is then

$$L_i = \frac{R_{12}}{2}(v_2 \sin(\theta_2 - \theta_{12}) - v_1 \sin(\theta_1 - \theta_{12})). \quad (14)$$

Again use the Point-n-Click program on your image to digitize the vectors \vec{R}_{12} , \vec{v}_1 and \vec{v}_2 . Use the earliest frame to determine \vec{R}_{12} . For best accuracy, use the first and last frames prior to the collision to determine the velocities, remembering to divide by the corresponding time interval to convert the measured distances into velocities.

When using the angles of your measured vectors in eq. (14) be careful about the signs of the sines: $\sin(\theta_1 - \theta_{12})$ and $\sin(\theta_2 - \theta_{12})$ are both positive or both negative.

Evaluate L_i and report

$$\frac{\Delta L}{L} = \frac{L_i - L_f}{L_i} \quad (15)$$

as your measure of the accuracy of conservation of angular momentum in the collision.

7.3.2 Conservation of Energy

The initial kinetic energy is given by

$$E_i = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}v_1^2 + \frac{1}{2}v_2^2, \quad (16)$$

using the convention that the pucks have unit mass. The final kinetic energy is the sum of that of the center-of-mass motion of the two-puck system plus that of the rotation about the center of mass (recall eq. (51) of Lab 5). Thus

$$E_f = \frac{1}{2}(2m)v_{cm}^2 + \frac{1}{2}I\omega^2. \quad (17)$$

The center-of-mass velocity is given by

$$m\vec{v}_1 + m\vec{v}_2 = 2m\vec{v}_{cm}, \quad (18)$$

so

$$v_{cm}^2 = \frac{1}{4}v_1^2 + \frac{1}{4}v_2^2 + \frac{1}{2}v_1v_2 \cos(\theta_1 - \theta_2), \quad (19)$$

using $m = 1$.

Evaluate v_{cm}^2 using the vectors \vec{v}_1 and \vec{v}_2 that you have previously digitized. Report E_f/E_i as a measure of the lack of energy conservation in the inelastic collision of the Velcro pucks.

7.3.3 Alternative Graphical Analysis

If you wish, you could substitute the following procedures for those of parts 7.3.1 and 7.3.2. Use the printout of your image of the Velcro puck collisions, a ruler with a millimeter scale and a protractor. Measure the lengths d , R_{12} , v_1 and v_2 with the ruler, and the angles θ_i , θ_{i+n} and the differences $\theta_1 - \theta_{12}$ and $\theta_2 - \theta_{12}$ with the protractor. Divide by the time interval where appropriate to obtain the velocities. The center-of-mass velocity v_{cm} can be measured graphically between the midpoints of the lines of centers of the pucks in two different frames. You now have all the ingredients to evaluate eqs. (8-10), (14), (16) and (17).

Another variation makes it simpler to evaluate eq. (14) by using eq. (12) directly. Use the result that the magnitude of $\vec{r}_1 \times \vec{v}_1$ can be written b_1v_1 where b_1 is the shortest distance from the center of mass of the two-puck system (in frame 1) to the line of vector \vec{v}_1 . Of course, this shortest line is perpendicular to vector \vec{v}_1 . Similarly the magnitude of vector $\vec{r}_2 \times \vec{v}_2$ is b_2v_2 . The signs of the two components of L_i are always opposite.

Please feel free to create your own analysis procedures...