Could Electrons Be Black Holes?

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In classical physics it is often convenient to suppose that electrons can be regarded as "point" particles.

In Einstein's (classical) theory of general relativity a "point" particle of mass m would actually be a black hole with apparent radius,

$$R = \frac{2Gm}{c^2}\,, \tag{1}$$

the well-known Schwarzschild radius, where G is Newton's gravitational constant, and c is the speed of light in vacuum (and in flat spacetime).

Hawking [1] added that quantum effects lead to radiation by a "black" hole, such that it has a surface temperature,

$$T = \frac{\hbar g}{2\pi ck}\,, \tag{2}$$

where g is the acceleration due to gravity at the surface of the black hole, and k is Boltzmann's constant.

Here, we estimate the lifetime of a black hole due to Hawking radiation, a quantum effect, supposing the black hole to be a thermodynamic black body, with surface gravity,

$$g = \frac{Gm}{R^2} = \frac{c^4}{4Gm}\,. \tag{3}$$

Then, the surface temperature (2) is,

$$T = \frac{\hbar c^3}{16\pi kGm}\,, \tag{4}$$

and according to the Stefan-Boltzmann law, the rate of radiation of energy/mass is,

$$\frac{dm}{dt} = \frac{1}{c^2}\frac{dU}{dt} = -\frac{4\pi\sigma R^2 T^4}{c^2} = -\frac{\sigma c^6\hbar^4}{(16\pi)^3 k^4 G^2 m^2} = -\frac{c^4\hbar}{960\pi G^2 m^2} \equiv -\frac{A}{m^2}\,, \tag{5}$$

noting that $\sigma = \pi^2 k^4/60\hbar^3 c^2$. Then, $m^2\,dm = -A\,dt$, $m^3 = m_0^3 - 3A\,t$, and the lifetime τ is,

$$\tau = \frac{m_0^3}{3A} = \frac{320\pi G^2}{c^4\hbar} m_0^3 = 5.3\times 10^{-18}\, m_0^3\ \text{s} = 1.3\times 10^{66}\left(\frac{m_0}{m_\odot}\right)^3\ \text{yr}, \tag{6}$$

for m_0 in kg, with the mass of the Sun being $m_\odot = 2\times 10^{30}$ kg.[1]

For an electron of mass $m_0 = 9\times 10^{-31}$ kg, its lifetime would be only about 4×10^{-108} s if it were a black hole.

That is, electrons are `not` black holes.

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[1] This estimate is similar to that in [2].

References

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