The Shape of Raindrops

They are not handsomely tapered but often resemble a small hamburger bun. This unpoetical form, frozen by high-speed photography, is analyzed to reveal the forces that mold it

by James E. McDonald

One of the few predictions that a meteorologist can make with confidence of almost 100 per cent accuracy is that if you ask an illustrator or cartoonist to draw a falling raindrop, his picture will be dead wrong. Without fail he will give it the streamlined shape commonly known as the teardrop. Meteorologists have known for many years that a real raindrop bears no resemblance to this drawing-board impostor. As pictured by high-speed photography, the usual small-sized drop (less than a millimeter in diameter) is almost perfectly spherical, and a larger drop is a squat object resembling nothing so much as a hamburger bun.

While this real picture is esthetically less satisfying than the teardrop fiction, it is of considerable interest to meteorologists. Just why the large raindrops take the deformed shape that they do has been a puzzle for half a century. I became interested in the problem in a casual way which seems to illustrate how the non-systematic approach in science can sometimes bear fruit. While browsing through a work on surface physics, I came upon a general equation which described the internal pressure in any fluid object, whatever the shape of its surface curvature. A slightly different equation, applying only to a spherical shape, had previously been tried on the drop problem without complete success. This more generalized relation was immediately recognizable as the key to an understanding of the deformation of large raindrops. It took only a few minutes to formulate the general outlines of a new hypothesis for raindrop shape. But to obtain an adequate check on this new hypothesis and to expand it in certain necessary details required several months' work. The results seem to show that one can, in fact, give a fairly good account of the way in which oversized raindrops develop their hamburger-bun shape. They also bring out that a falling raindrop is the seat of some surprisingly complex physical processes.

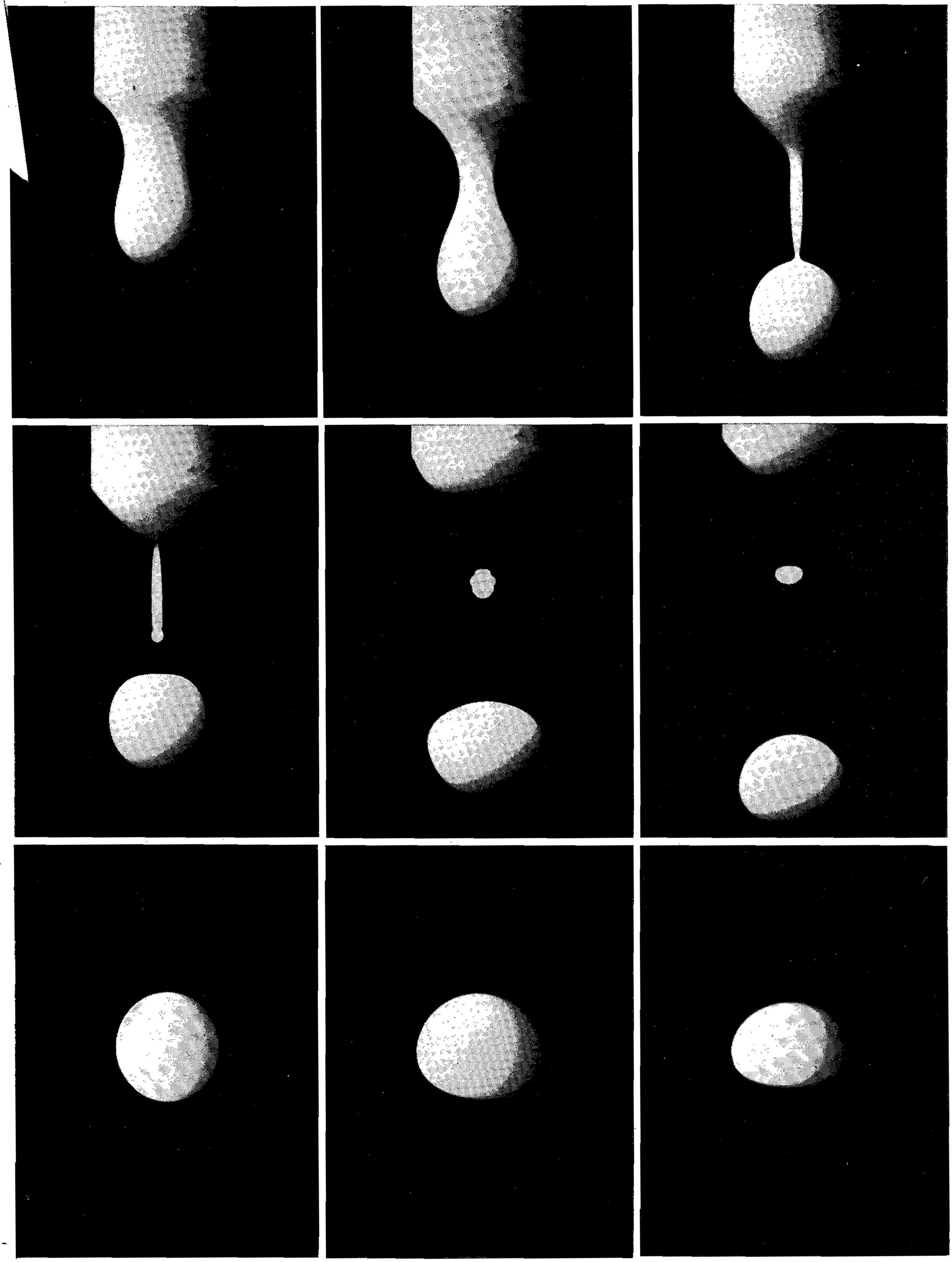
One might begin by asking why a drop of water should bear any resemblance to a sphere at all. The answer is that surface tension always tends to reduce the surface of a free mass of liquid to the smallest area it can achieve. The smallest possible surface area is that of a sphere, and an isolated drop of liquid not distorted by external forces is pulled by its surface tension into a spherical shape. In terms of thermodynamics, it adjusts itself to the spherical shape to minimize its surface free energy.

The question can also be considered in terms of pressure. The internal pressure just inside the convex surface of a drop is higher than the external pressure prevailing in the surrounding gas. The smaller the radius of curvature, the greater is the pressure difference between the two sides of the convex surface. (For example, a cloud droplet of one micron radius has an internal pressure of more than two atmospheres.) If our isolated drop should momentarily assume some shape other than that of a sphere, its surface would have different radii of curvature at different points, and the internal pressure just below the surface would momentarily be dissimilar at these various points. The consequent pressure gradients within the drop would tend to force liquid from the regions of sharp surface curvature to those of more gentle curvature. This is equivalent to saying that surface tension, through its control of internal pressure, reshapes the drop into a sphere whenever it happens to become slightly deformed. When the drop is finally brought into the shape of a perfect sphere, the uniform surface curvature makes the pressure difference uniform at all points of its surface, and the internal pressure within the drop also is uniform, provided that the external pressure field around the drop remains so.

Such a uniform pressure field actually exists in an ordinary fog or cloud, and, sure enough, photomicrographs of cloud and fog drops show that these tiny particles are indistinguishable from perfect spheres. Doubtless precise measurements would disclose that they depart slightly from perfect sphericity, because of the weak gravitational and aerodynamic forces acting on them, but a manufacturer of precision ball-bearings would be very happy to turn out bearings as close to perfection as these drops.

The disturbing effects on drop shapes that appear as one considers larger and larger drops seem to be due almost entirely to aerodynamic and gravitational forces. A. F. Spilhaus, now at the University of Minnesota, was the first to call attention to the role played by aerodynamic forces in shaping raindrops. As drops grow larger and their falling speed increases, the disturbance of the air sets up momentary non-uniformities in the air pressure around them. The big drops that pelt down in a summer thunderstorm plummet through the air so fast that they continuously create about them their own shape-deforming pressure fields as they fall.

Now it is well known that when a body falls the air pressure just under the body becomes higher than average and the pressure around its sides lower than average. This means, of course, that the internal pressures within a large falling



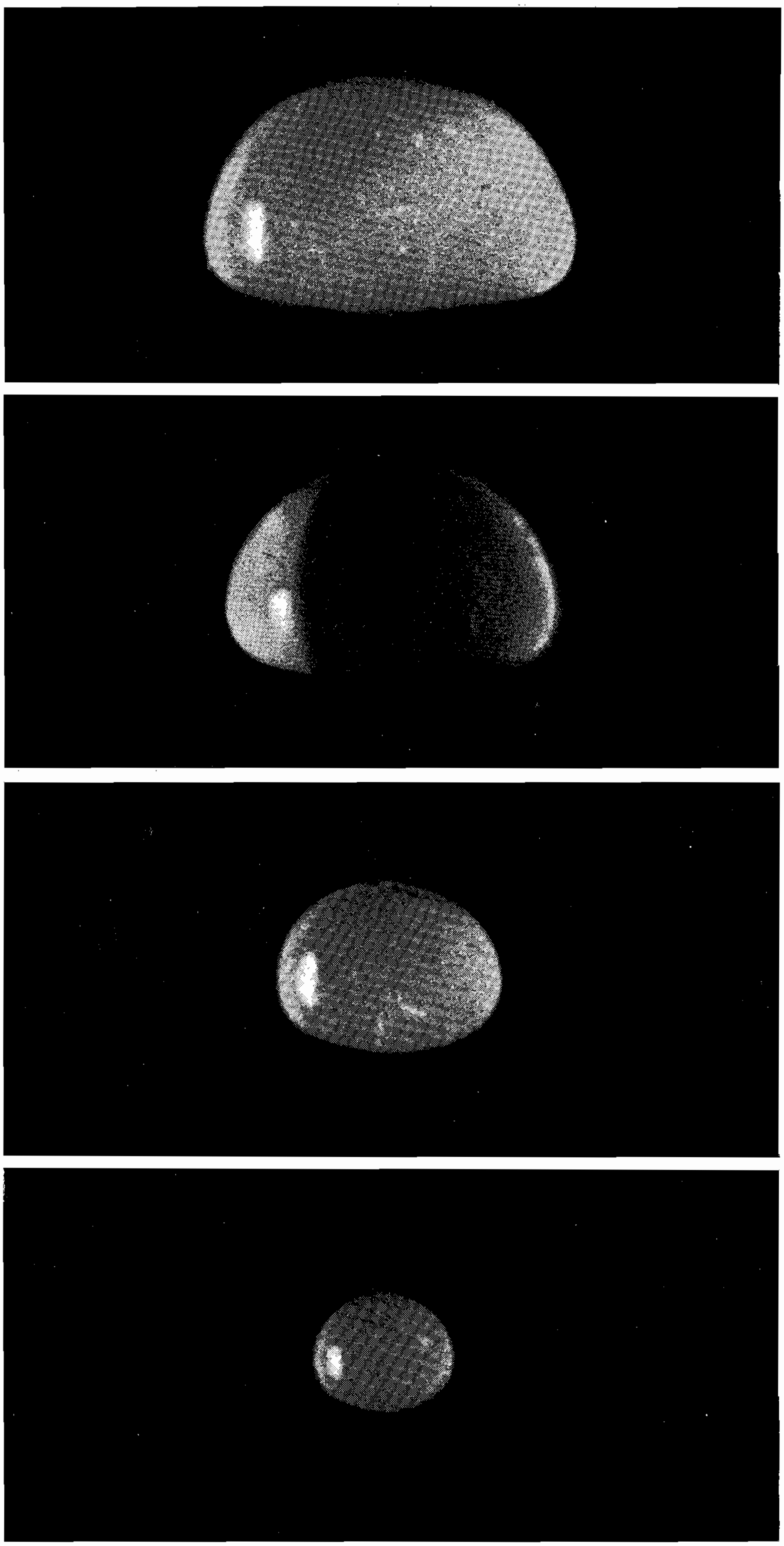
FALLING DROPS OF MILK, used instead of water because of their high visibility, were photographed with high-speed flash by Harold E. Edgerton of the Massachusetts Institute of Technology. In the third, fourth and fifth pictures the shape of the drop briefly oscillates. In the sixth it is spherical. In the last picture the drop has attained an almost stable configuration after a fall of 14 feet.

raindrop must change accordingly; the drop develops an excess of pressure near its bottom and a deficiency of pressure all around its waist. And if we momentarily assume that the air flow is that of a perfect fluid, there will be an excess of internal pressure near the top of the drop as well as near the bottom.

At this point we obtain a vivid notion of how a drop takes care of its own shape. The gradients of internal pressure drive water from near the base and top out into the regions around the waist, thereby tending to flatten the drop and increase its horizontal diameter. Even more intriguing is the fact that the resultant modification of the drop's surface curvature is of just the kind required to help the drop restore a uniform internal pressure and achieve an equilibrium. The sharpening of the curvature around the waist adjusts the surface tension effects to make up for the deficiency of external pressure there, while the flattening of curvature near the base and top tends to cancel the effects of the excessive external pressures in those regions. Together the joint action of surface tension and aerodynamic forces deforms the drop continuously until it reaches a stable internal pressure distribution.

But clever as a raindrop may now seem in managing its affairs, one must ask whether a large drop has truly brought itself into complete mechanical equilibrium when its internal pressure is uniform. The answer is that it has not, for it must still meet an important demand of the laws of gravity: that is, it must develop a vertical pressure gradient just sufficient to permit the lower strata of the drop to hold up the upper ones in the gravitational field. Briefly, a liquid drop falling at terminal velocity can be in full mechanical equilibrium only when its internal pressure, instead of being uniform throughout, varies vertically in such a way as to satisfy the familiar hydrostatic equation relating liquid density, liquid depth and the acceleration of gravity. If the drop were accelerating freely in the earth's gravitational field, this hydrostatic requirement would not appear. But raindrops reach a terminal uniform velocity after only a few yards of fall; hence small but important hydrostatic pressure gradients must exist within them.

In the tiny droplets of clouds the hydrostatic gradients are not important, because the difference in internal pressure from top to bottom of these microscopic globules is negligibly small compared to their internal pressure increase



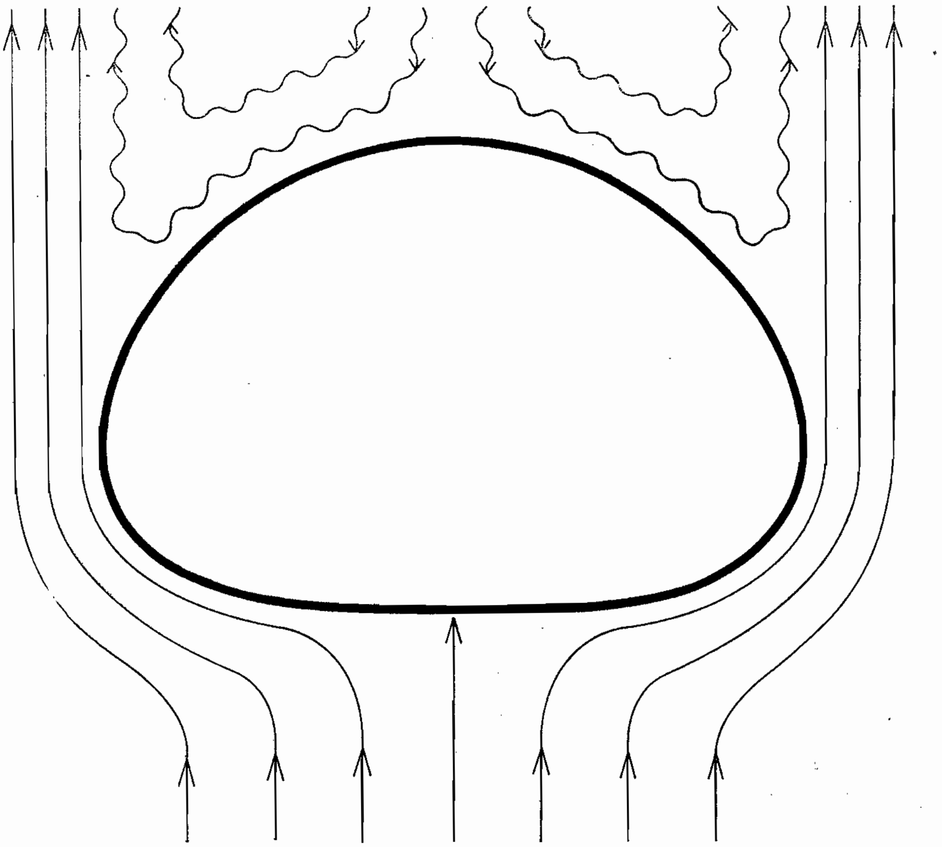
FALLING DROPS OF WATER were photographed by Choji Magono of Yokohama National University. The volume of the drop at the top is equivalent to that of a sphere 6.5 millimeters in diameter; its velocity is 8.9 meters per second. The corresponding numbers for the other drops are as follows. Second drop: 6 mm. and 8.8 meters per second. Third: 4.8 mm. and 8.3 meters per second. Fourth: 2.8 mm. and 6.8 meters per second.

due to surface tension. The top-to-bottom hydrostatic pressure differential varies directly with drop size, while the surface pressure increment varies inversely with drop size. Consequently, by the time a cloud droplet has grown into a large raindrop hydrostatic effects have become about equal in importance to surface-tension effects—a point which appears to have been overlooked by those who have examined the raindrop-shape problem in the past.

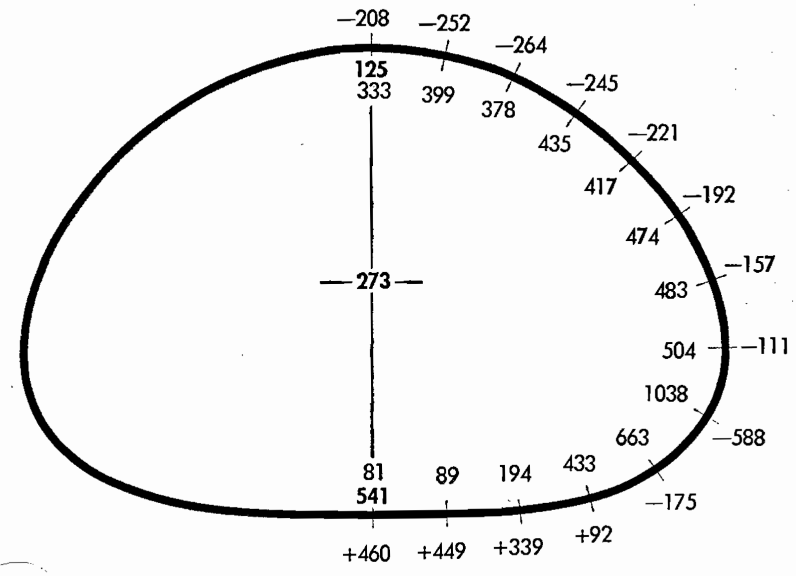
Combining the hydrostatic principles with the aerodynamic principles, one next obtains a curious result. If we demand that drops deform in such a manner as to yield an internal pressure field satisfying the hydrostatic equation, we find to our embarrassment that the drop must be flatter on its upper side than on its lower—which is just the reverse of the shape that falling raindrops actually take.

An aerodynamicist would spot the difficulty very quickly. We have assumed so far that the drops are falling through a perfect, non-viscous fluid. Actually air has some viscosity, enough to have a significant effect on an object of raindrop size (one to five millimeters in diameter) and falling speed (five to eight meters per second). Around a large raindrop the air must behave essentially as it does over the wing of an airplane in a stall: the boundary layer of air just next to the object (raindrop or aircraft wing) separates from the object and leaves a turbulent wake. In such a wake region the air pressure is always lower than it would be for perfect fluid flow in which the streamlines neatly close in behind the object. Thus the lower air pressure in the wake of a falling drop forces a greater curvature of its upper surface than of its underside.

When the drop has accomplished this adjustment, it is at last in full equilibrium with all of the important forces that play upon it as it cleaves through the air. It seems almost unfair that the fate of so cleverly equilibrated a little system may be no more glorious than to splatter down on some dusty road at the beginning of an August thundershower.

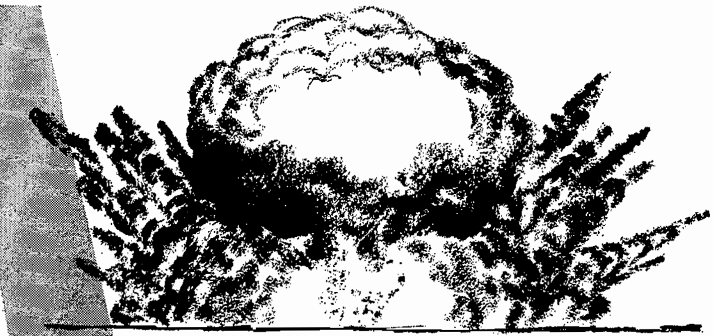


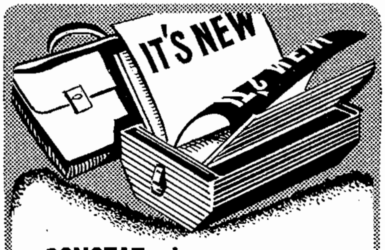
FLOW OF AIR around a large falling raindrop is indicated by the red lines in this diagram. The boundary layer streamlines follow the curve of the drop until they reach the "separation point." Above the drop and enclosed by the separating boundary layer is a turbulent region. The low pressure of this region is responsible for the shape of the drop.



DISTRIBUTION OF PRESSURE is given for a 6-mm. drop falling 8 meters per second. The numbers outside the drop give the difference between the pressure around the drop and that of the atmosphere. The red numbers inside give the difference in surface pressure; the black numbers, in internal pressure. The units are dynes per square centimeter.

The role of boundary-layer separation came to light only after I had computed pressure profiles from measurements made on an actual photograph of water drops provided by Choji Magono, now of Yokohama National University. The method used to deduce the aerodynamic pressure distribution over the surface of a drop hinged upon the use of certain relationships concerning the differential geometry of surfaces of revolution obligingly worked out by my colleague J. M. Keller at Iowa State College. The falling speed and size of the photographed drop were known, and its curvature at every point could be accurately measured on the picture. The first step in the analysis was to determine the pressure at a single point. Fortunately the pressure at the center of the underside of a falling body can be calculated even if nothing is known about the rest





of the airflow pattern. This, added to the pressure due to surface tension which was computed from the curvature at the bottom, gave the total pressure just inside the lowest point of the drop. Since the drop was in hydrostatic equilibrium, the internal pressures at all the other levels could be calculated. The surface tension pressures at other points were calculated from curvature measurements and subtracted from the internal pressure. This gave the outside air pressure all around the drop. The pressure pattern thus deduced turned out to be much like the patterns observed in wind-tunnel work on separating boundary layers.

The idea that there was a separation of boundary layers around raindrops agreed with observations. Ross Gunn of the U. S. Weather Bureau had reported a curious sideslipping of falling drops of about one millimeter diameter, and he had shown rather convincingly that this odd behavior must involve eddies. Eddies can be shed only from a wake region enclosed by a separating boundary layer. Later a more quantitative type of evidence for the shape hypothesis was obtained by means of a calculation of the total pressure drag acting on a falling drop. In the case of a drop falling with uniform velocity, this pressure drag plus the drag of friction (which is very small) must be equal to the weight of the drop. It was found that the drag was in fact equal to the weight to within the limits of precision of the methods employed in the pressure calculation. It now appears reasonably safe to conclude that the queer shape of a large raindrop results in an understandable way from a conspiracy among the forces we have here examined.

Of what use is the result? First of all, it is always pleasant to acquire some understanding of even a minor peculiarity of nature. Then also, the study yielded information which may be useful for solving the vexed problem of why and how raindrops break into fragments in the turbulent regions of clouds. Finally, the clear recognition of the role of boundary-layer separation in the aerodynamics of raindrops will almost certainly help to clarify the nature of heat and vapor transport at the surface of falling raindrops.

But, to end on a note of dark pessimism, it seems quite improbable that any amount of progress in exploring the drop-shape problem will persuade cartoonists and commercial artists to alter the shape of their peculiar brand of raindrops.