

Laser Beams and Resonators: Beyond the 1960s

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Invited Paper

Abstract—This paper describes the continuing advances in laser resonators and optical beam propagation that emerged in the decades following the 1960s, growing out of the fundamental concepts from that era reviewed in an earlier paper. It also presents a brief look forward at some of the continued innovations now emerging in these areas.

Index Terms—History, laser beams, optical resonators.

I. INTRODUCTION

ALL OR nearly all the basic concepts associated with real stable and unstable optical resonators and with the propagation of real Gaussian beams were in place by the mid to late 1960s, as described in the first of this pair of reviews [1].

The development of new concepts related to resonators and beam propagation nonetheless continued at a rapid pace through the following three decades, and continues even today, motivated by new kinds of lasers, by requirements for more complex and multielement laser resonators, by considerations of laser beam propagation in practical applications, and by a continuing desire for basic understanding of optical beams. This second paper attempts to provide an overview of the most important of these further advances from the late 1960s onward.

II. BEYOND THE 1960S: “COMPLEX AND TWISTED”

A significant fraction of the developments in optical resonators and beam propagation from the late 1960s onward can be described as “complex,” “twisted,” and even “nonnormal” in character, to use only slightly hyperbolic terms whose meanings will emerge in the following sections.

A. Complex and Twisted Paraxial Optics

During the late 1960s, laser researchers became increasingly interested in gain-guided lasers, along with lasers having transversely tapered internal apertures and laser mirrors having transversely tapered reflectivity. It was soon realized that all these systems could be described using existing analytical tools simply by converting most of the real-valued parameters in Gaussian beam theory into complex-valued quantities.

At about the same time, some optics researchers also became interested in optical structures characterized by general astigmatism or “twist” about the axis of propagation. This was moti-

vated in part by the development of nonplanar ring resonators as described below, along with optical isolators or optical diodes employing Faraday rotation.

The intellectual history of these combined developments beginning in the late 1960s is nearly as “complex and twisted” as the topics themselves. The basic ideas emerged in overlapping papers from many different authors, making it difficult to trace primary origins in the extensive literature that has developed. The following summary, therefore, cannot always identify just which individuals contributed which concepts.

B. Complex Ducts and Complex Gaussian Beams

The concept of an elementary Gaussian beam as a spherical wave diverging from a complex-valued source point emerged early in the laser era, as did the recognition that one could combine the wavefront radius of curvature R and Gaussian spot size w into a single complex-valued Gaussian \tilde{q} parameter defined by $1/\tilde{q} \equiv 1/R - j\lambda/\pi w^2$. The conceptual relationship between narrow optical beams and complex-valued eikonal rays was also recognized in more general approaches to optical propagation [2]–[6].

Early discussions of propagation through parabolic gain-guided or “complex parabolic” lensguides were given in 1965 by Kogelnik [7] and a few years later by Casperson and Yariv [8]. A slightly later discussion of laser resonators having transversely varying mirror amplitude reflectivity (VRM resonators) was published by Zucker [9]. The author also made an early suggestion for using Hermite–Gaussian modes with complex argument as free-space eigenfunctions [10].

Further advances came as laser workers realized that an aperture with a parabolic transmission profile was formally equivalent to a thin lens with an imaginary focal length, and a mirror with a parabolic reflectivity profile was formally equivalent to spherical mirror with an imaginary-valued radius of curvature. More generally, systems with parabolic transverse gain or loss profiles were recognized as formally equivalent to optical fibers or ducts having imaginary-valued transverse index variations.

Essentially all the formal mathematical results for real-valued optical systems could then be converted to handle these complex-valued systems simply by adopting complex values for the ABCD matrix elements [11]–[13], the Gaussian spot size w , and the arguments of the Hermite–Gaussian and Laguerre–Gaussian functions. One can refer to these complex-parabolic systems in general as complex Gaussian ducts and their eigenmodes as complex Gaussian beams, or if necessary one can speak of generalized complex-valued ducts

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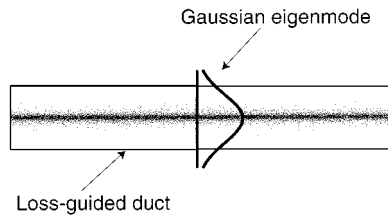


Fig. 1. Schematic drawing of the lowest order complex Gaussian beam in a loss-guided or gain-guided duct. The lowest and higher order modes have diverging phase fronts (not shown), but maintain fixed intensity profiles.

for those more general cases where the transverse variations are not simply parabolic [14]–[18].

Fig. 1 is a schematic illustration of a complex Gaussian beam with a fixed amplitude profile propagating in a complex duct. A particularly important aspect of complex paraxial optics is that systems with positive gain or loss guiding can still support transversely confined and guided Gaussian modes even in the presence of divergent or unstable mirrors, as demonstrated by Casperson and Yariv [8]. (Note that the essential point is that the system have higher transmission on axis and decreasing transmission off axis, whether this is accomplished by radially decreasing gain or radially increasing attenuation.)

Casperson also appears to have been the first to note that there was an important distinction between transverse confinement and perturbation stability for complex Gaussian systems [19]–[23]. Complex Gaussian concepts were also applied to one-dimensional modes in gain-guided semiconductor diode lasers [24], to free-space propagation and to other forms of complex Gaussian resonators [25]–[30], and to the propagation of partially coherent light [31]–[33]. Additional work along these lines has also continued into the past decade [34]–[42].

The variable reflectivity mirror (VRM) concept which was pioneered by Zucker [9], used in combination with a geometrically unstable resonator design [43], provides perhaps the best current design approach for obtaining the desired combination of large mode volume, moderate output coupling, and good mode discrimination and beam quality in higher power and higher energy lasers [37], [38], [44]–[51]. This approach does, however, remain somewhat hampered by difficulties in obtaining the necessary VRMs.

C. Twisted Beams and General Astigmatism

For the elementary case of an optical system having ordinary astigmatism, one can simply make a separation of variables along the transverse principal axes of the system, and apply ordinary Gaussian beam arguments (complex valued, if appropriate) separately along the resulting orthogonal x and y axes.

Suppose, however, that one assembles an optical resonator or lensguide containing astigmatic or anisotropic paraxial elements, for example, cylindrical lenses and Brewster plates, with the principal axes of individual simple astigmatic elements rotated about the system axis by various angles other than multiples of 90° . The result is a general astigmatic system, or an optical system with an inherent “twist.” Twisted optical systems of this sort have nonorthogonal principal axes and their transverse eigenmodes are characterized by image rotation and polarization rotation properties.

Optical beams propagating in free space can also possess an inherent “twist” independent of any optical system (although some sort of twisted optical system may be necessary to create a twisted free-space optical beam). Even in a lowest order Gaussian beam, for example, the principal axes of the Gaussian intensity profile may be rotated by an arbitrary angle from the principal axes of the toroidal rather than spherical phase profile. The principal axes for the intensity profile of such a beam may then rotate by up to 180° as the beam propagates from $z = -\infty$ to $+\infty$. Such beams will then no longer have a unique waist, or even two unique orthogonal waists where the transverse spot sizes are minimum and the wavefront curvatures simultaneously flat. One can also create twisted optical beams in which the intensity profile does not visibly twist but in which there is a twisted or helical character to the phase profile within the beam [52]–[57], leading in general to an increased diffraction spreading for the beam.

Recent decades have brought forth extensive discussions of twisted optical systems and twisted optical beams. Much of the early literature in this area is associated with nonplanar ring resonators as described elsewhere in this paper. Of note are publications by Arnaud [5], [52], [58]–[63], and much fundamental work on the classification and analysis of stigmatic, simple astigmatic, and general astigmatic beams and systems has been done by Nemes [64]–[70].

In the most general case, where the optical system and the propagating beam may have both complex-valued and twisted (general astigmatic) character, the ray matrices [71] must be expanded from four potentially complex ABCD coefficients (actually three independent elements) to four complex-valued two-by-two tensor coefficients \mathbf{ABCD} [52], [55], [57] with up to ten independent elements [65], [67]–[69], [72]. The complex-valued Gaussian \tilde{q} value similarly becomes a complex-valued tensor quantity $\tilde{\mathbf{Q}}$ [12], [58], [73], [74]. Methods for transforming the properties of general astigmatic beams in various ways using general astigmatic systems are of interest [66], [75] as are the angular momentum properties of twisted beams. Additional papers in this area include [54], [76]–[85]. A recent overview of optical twist has been given by Friberg [86].

III. ADDITIONAL ADVANCES BEYOND THE 1960S

A. Numerical Beam Propagation: Fast Transforms

Fox and Li’s original calculations employed straightforward numerical integration of the Huygens’ integral for N sample points across each transverse dimension. The same propagation calculations can also be carried out using Gaussian mode expansions [87], but both of these methods are computationally intensive and sensitive to round-off errors. From another viewpoint, Huygens’s integral in scalar form is a convolution integral which can be evaluated using Fourier transform methods [88], [89]. The required Fourier transforms for a uniformly sampled beam profile can then be converted to discrete Fourier transforms, which can be evaluated using the fast Fourier transform (FFT) algorithm. The merits of this FFT approach include not only reducing the N^2 computational problem for one transverse dimension to an $N \log N$ problem, where N is the number of sample points needed to accurately describe the beam profile,

but also substantial reductions in required computer storage and especially in round-off errors.

The first applications of the FFT algorithm to optical resonator and beam calculations were by Sziklas and Siegman [90], [91] and by Johnson [92], and this has now become the standard approach for commercial propagation codes, for example those of Lawrence [34], [93]–[95]. A quasi-“fast Hankel transform” (FHT) algorithm which uses the FFT algorithm internally and captures many of its advantages, was also developed a few years later for propagation calculations in cylindrical coordinates [34], [96]–[98].

B. Finding Higher Order Modes: The Prony Method

The Fox and Li method in its simplest form eventually converges to the lowest order or lowest loss eigenmode of a resonator, but convergence can be slow if there are multiple modes with very similar losses. In some cases, one can also extract individual higher order modes, for example, with the convergence method illustrated in [1, Fig. 2], or with the resonance excitation approach used in later calculations by Fox and Li [99]. Powerful numerical algorithms for extracting all the eigenvalues of large complex matrices can also be applied to the resonator problem [100].

The general problem in any case is to fit the results from repeated round trips inside the resonator with a superposition of exponentially decaying complex-valued eigenmodes. This problem arises in other situations as well, and is known to be mathematically difficult. One relatively simple procedure for fitting N round trips exactly with N decaying exponentials was invented by the Baron de Prony in 1795 [101] and introduced serendipitously to the author by Tuttle [102] in the late 1960s. This Prony method has since been applied to the calculation of optical resonator eigenmodes by a number of workers [103]–[108].

It is also possible to apply the Fox and Li approach to nonlinear and multimode problems, for example, competition between transverse modes in a laser oscillator. Klein and coworkers [109] accomplished this by carrying forward two simultaneous but independent Fox and Li calculations representing two possible modes inside the same resonator, together with a gain sheet that saturated based on the total intensity profile of both modes. They observed that, in accurate mimicry of a real laser, under some circumstances one array built up to a dominant mode while the other decayed to noise level as a result of mode competition, whereas in other cases both arrays built up to steady-state levels in two different transverse modes. The same technique has subsequently been used by others [110].

C. Operator Methods and Quantum Analogies

A Gaussian beam propagating off axis in a parabolic index profile is formally equivalent to a Gaussian quantum wave packet in a parabolic potential well. A number of workers have exploited the close formal analogies between paraxial beam propagation and quantum theory by applying quantum operator techniques and algebraic methods as alternative approaches to paraxial beam and resonator problems [26], [27], [111]–[122].

D. Nonnormal Modes

As first noted by Fox and Li, the transverse eigenmodes of an open-sided or gain-guided lensguide or resonator are not power orthogonal in the usual sense and thus do not form the usual set of “normal modes” characteristic of most other linear systems. Rather, the resonator eigenmodes are biorthogonal to a set of adjoint modes which correspond physically to eigenmodes propagating in the reverse direction along the same waveguide or around the same resonator [63], [123]–[128]. The purely real Hermite–Gaussian or Laguerre–Gaussian functions that are widely used to analyze elementary stable cavities do form complete and orthonormal sets, but these functions are only approximations (though often extremely good approximations) to the slightly nonorthogonal exact eigenmodes in these cavities. The complex Gaussian transverse modes in complex Gaussian laser cavities and also the eigenmodes of unstable resonators are distinctly nonorthogonal in character. As a general principle, purely index-guided systems such as conventional optical fibers have normal modes, but gain-guided, loss-guided, or finite open-sided systems have nonnormal modes.

The nonnormal character of optical resonator eigenmodes leads to a number of nonstandard and quite nonintuitive mathematical and physical properties for these systems. For example, the total power or energy in a nonnormal lensguide or resonator is not simply the sum of the powers or energies in the individual modes, even if the eigenmode functions are individually normalized. In addition, maximum excitation of a given eigenmode by an externally injected signal is obtained not with the usual matched coupling but with so-called “adjoint coupling” [42], [129]–[133]. Injecting a reversed or adjoint wave into a nonnormal system, as shown for a gain-guided duct in the upper part of Fig. 2, causes the lowest order eigenmode to be excited with more power or energy than the input wave itself, as shown by the extrapolated portion of the $n = 0$ line in the lower part of Fig. 2. This adjoint coupling to the lowest mode is necessarily accompanied by simultaneous excitation of other higher order eigenmodes which then die out relative to the lowest order mode.

The second quantization procedure that is widely used for quantizing the electromagnetic fields of a laser oscillator in most fully quantum laser analyses [134] is also called into serious question in nonnormal optical resonators [132], [133], and the procedure for expanding the fields in an optical resonator or lensguide in terms of its eigenmodes must also be substantially modified from the usual least-squares normal-mode expansion [29], [42]. The Rayleigh Ritz procedure used for estimating eigenvalues in many other systems also cannot be applied to optical resonator modes [135].

Finally, laser oscillators having distinctly nonorthogonal modes will experience a substantial excess spontaneous emission per mode or excess quantum noise factor, leading to a corresponding increase in their Schawlow–Townes linewidth. This excess noise effect was first proposed by Petermann for gain-guided diode lasers [136], and has since been shown by the author to be entirely determined by the nonnormal or nonorthogonal character of the eigenmodes [137]–[139]. Large increases in the Schawlow–Townes linewidth in otherwise

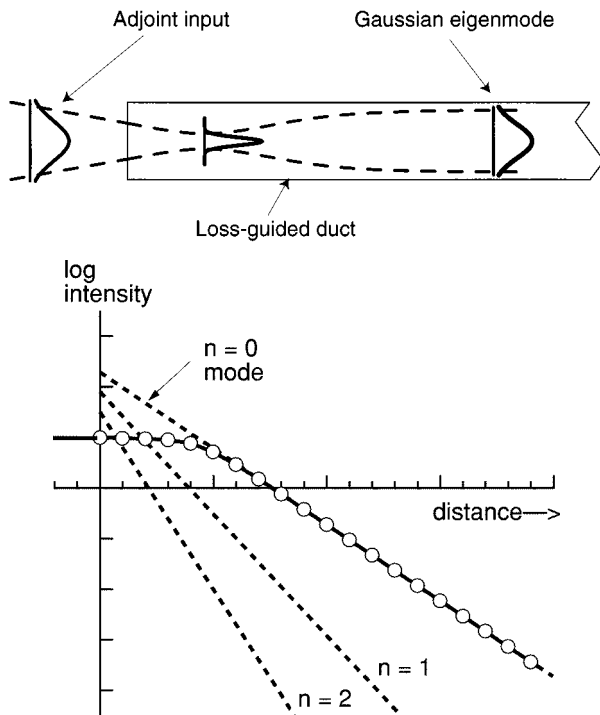


Fig. 2. Maximum excitation of the lowest order eigenmode in a complex Gaussian duct is obtained not by the usual mode-matched coupling, but by so-called adjoint coupling.

conventional lasers with nonorthogonal transverse modes have been confirmed by groups at Stanford [140]–[143], Leiden [144]–[146], and Rennes [147]. Recent work has shown that the polarization eigenmodes in certain twisted-mode cavities are also nonnormal or nonorthogonal in polarization space, leading to similarly increased quantum noise properties when used as laser oscillators [145], [148], [149].

E. Nonplanar Ring Resonators

A ring resonator having four or more mirrors can be converted into a nonplanar ring resonator by warping the ring, for example, by lifting any one of the mirrors out of the plane formed by any three of the other mirrors. At some time in the early 1960s it was realized that as an optical beam travels around such a nonplanar ring the beam profile will rotate or twist about the direction of propagation by an amount that depends on the nonplanarity of the ring. The polarization axes for the beam will also rotate about the propagation axis by a related amount. Both effects are manifestations of Berry's geometrical phase [150].

Early papers on nonplanar ring resonators for lasers include those of Danileiko and Lobachev, Popov, Arnaud, and subsequent authors [59]–[61], [151], [225], [152]–[157]. The image rotation aspect of these resonators has led to a useful class of unstable ring resonators, sometimes called "UR90" resonators [158]–[168], in which the image rotation around the ring path is 90° and the output beam emerges as a single completely filled rectangular beam, in contrast to the annular output beam from a conventional unstable resonator.

The polarization rotation properties of nonplanar rings have been employed to create more complex types of ring laser gyroscopes [169]–[173]. They have also been used in conjunction

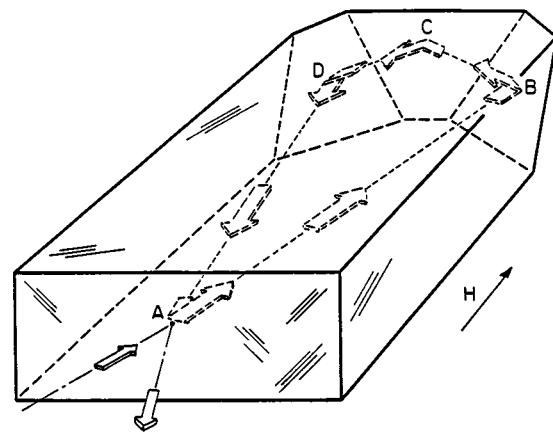


Fig. 3. Monolithic nonplanar ring oscillator (NPRO) [176].

with a small amount of Faraday rotation to create unidirectional ring laser oscillators [174], [175], notably the very compact and stable monolithic nonplanar ring oscillator (NPRO) of Kane and Byer [176], [177] shown in Fig. 3. Nonplanar rings also display modified misalignment or axis stability properties as analyzed by several authors [59], [178]–[181].

F. Lasers and the Sagnac Interferometer

The Sagnac interferometer, a.k.a. the "antiresonant ring," is another classic optical element that has found widespread applications in lasers. In addition to prism end reflectors [182], [183], it has been employed as a laser end mirror for laser cavity dumping and switching and for active and passive (CPM) mode locking [184]–[194]. With nonlinear elements inserted it can also be used for saturation spectroscopy and nonlinear optics [195]–[199]. The availability of fiber couplers and ring resonators has extended these concepts to fiber-optic implementations [200], [201]. Most recently the Sagnac interferometer has emerged as a promising concept for laser-interferometer gravity-wave observations (LIGO) [202]–[204].

G. Chaotic Resonators

Stable resonators and lensguides are designated as such in part because their ray paths, including even the off-axis ray paths, are stable against small perturbations in initial position or direction. There are elementary closed geometrical shapes or closed resonators, however, in which all the internal ray paths that might be taken by a bouncing light beam or a classical billiard ball are completely chaotic or perturbation-unstable in character. That is, no matter how close together two rays may start out in such a structure, their trajectories will eventually diverge in a chaotic fashion. The statistical properties of the rays and resonant modes in these chaotic structures are of substantial interest to workers in chaos and nonlinear dynamics.

It can be of interest, therefore, to see if there are any connections between this chaotic ray behavior and the oscillation modes that will be established in such a structure if the structure can be somehow filled with a laser gain medium. One widely discussed chaotic structure is the "stadium billiard" or stadion [205]–[208] which consists essentially of two cylindrical or spherical end caps forming a one-dimensional unstable optical

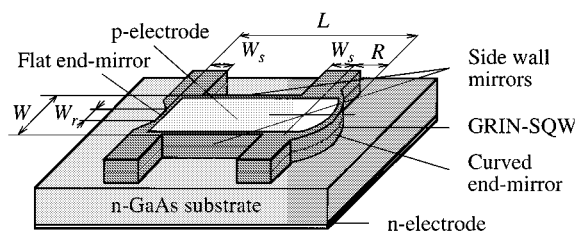


Fig. 4. Chaotic stadium resonator with internal gain created by reactive-ion etching on the surface of a GaAs diode laser.

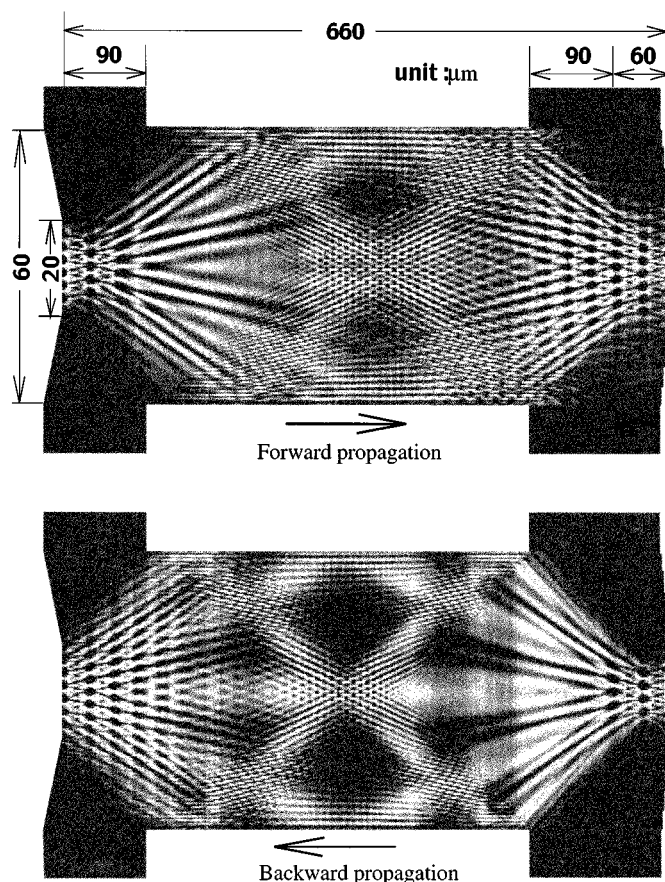


Fig. 5. Oscillation profile within the chaotic resonator of Fig. 4. The oscillating “scars” are different in the two directions along the structure.

resonator with side walls added. Preliminary experiments to study the oscillation behavior in such a stadium structure have recently been carried out by using reactive ion etching techniques to fabricate a “stadium laser” on the surface of a semiconductor laser, as shown in Fig. 4 [40], [209]. The resulting oscillation properties of this chaotic stadium-resonator laser, as shown in Fig. 5, can then be observed and explained in some detail using an extended Fox and Li analysis [40], [210]. There appear to be possibilities for further exploration of this subject [211].

H. Laser Arrays, Higher Order Modes, and Talbot Mirrors

The lowest order Gaussian mode in any conventional stable optical resonator has a mode diameter $d \sim (L\lambda)^{1/2}$ which for a length L on the order of a meter and a wavelength λ on the order

of one micrometer means a mode diameter on the order of a millimeter or less. Laser gain media are readily available, however, in the form of laser rods or gas laser tubes with diameters of centimeters or more. Extracting all the available laser power or energy from these large-volume gain media while maintaining single-mode operation and thus good beam quality can, therefore, be a difficult problem, not to mention the damage problems associated with handling very large powers or energies in small diameter beams.

The unstable optical resonator offers one useful solution to this dilemma, although not without creating other difficulties. Much effort has gone into the search for other approaches to high-power or high-energy large-mode-volume lasers. One approach is to not only allow but encourage laser oscillation in a single higher order (and thus larger diameter) cavity mode, and then to use some form of phase correction to convert the highly divergent but nonetheless fully coherent output beam from this laser into a more desirable beam profile. Laser oscillation in a single controlled higher order mode has in fact been achieved in several types of lasers, using either various types of modal filters within the laser resonator or variations on Talbot mirrors [212] to select a single oscillating mode.

Converting the resulting output beam into a truly high-quality beam is, however, a more uncertain proposition. Even if the near-field phase profile can be converted to an ideal uniphase wavefront so as to obtain a narrow central lobe in the far field, the transverse intensity variations inherently associated with a higher order mode mean that a large amount of the total beam energy may still reside in weak but numerous side lobes surrounding this main central lobe.

Another generic approach that comes naturally to those familiar with phased-array radars at microwave frequencies is to construct a large number of lower power small-diameter single-mode lasers, and then attempt to combine their beams into a single phased-array output. Much effort has gone into variations on this approach, using for example coupled laser oscillators with conventional cavities, or arrays of coupled waveguided lasers such as, for example, closely adjacent laser stripes on semiconductor diode laser chips.

One generic problem associated with this approach is that success requires that the individual oscillators operate not only all at exactly the same frequency but all with precisely controlled phases relative to each other—and not only are the dimensional tolerances involved in precise phase control much more severe at optical frequencies than at microwave frequencies, but the problems involved in obtaining adequately single-frequency operation are also much more difficult in lasers than in typical microwave devices. Locking together some significant number N of quasi-independent lasers requires either potentially herculean control system efforts, or some form of optical coupling between the individual lasers in order to lock together oscillators that must already be highly stabilized in frequency. Even if the latter can be achieved the generic situation is the N -coupled oscillators can potentially operate in any of N “supermodes” of the coupled system, only one of which is wanted, and the rest of which must be adequately suppressed or discriminated against—and even if this can be achieved the beam-quality or side-lobe problem mentioned above still remains.

A review of some of the ideas discussed in this section has been given by Pashinin [213]. I have not attempted to survey the very large literature devoted to the above topics, and to laser arrays in particular, in part because many of the publications are devoted to the complexities of array theory rather than modal concepts, in part because it is difficult to identify the seminal or key papers in the field, and in part because I have some concern that this general approach, despite the effort devoted to it, may not in the end provide the kind of results that are sought after.

IV. CONCLUSION AND A BRIEF LOOK FORWARD

My objective in these two reviews has been to provide a historical look back at some at least of the most important earlier and continuing advances in laser resonators and laser beam propagation during the past four decades. In doing this, I have slighted certain topics, including roof resonators and distributed-feedback and Bragg-mirror resonators, as mentioned in the first review. Among developments in more recent decades, I have also not attempted to review phase-conjugate resonators or resonators employing stimulated back-scattering, despite their interesting properties and extensive literature, in part because they fall more in the domain of nonlinear optics. I have only given general comments on the broad topic of coupled laser resonators and arrays. I have also not attempted to cover recent developments in the obviously very important and promising subject of fiber lasers.

It is traditional to conclude any such look backward in a given area with at least a brief look forward in the same area, and my own look forward will be brief. In addition to fiber lasers, particularly significant areas for continued progress in laser resonators and beams would seem to include at least the following topics.

A. Custom Resonator Designs

In the early years of the laser era, research efforts focused on understanding the modal properties of simple resonator structures. In recent years, the advent of more powerful computers, new ways of fabricating complex mirror shapes, and new concepts in tomography and adaptive optics have led to increased interest in the synthesis of customized resonator structures and especially of custom mirror profiles. The objective in these efforts is to achieve resonator modes having predetermined mode profiles, such as, for example, uniform or flat-topped mode profiles, which may be better suited to particular applications or to extracting more energy from a given laser medium.

Practical examples of this approach include mirrors with graded or nonspherical phase profiles to achieve individual passive "mode conjugation," graded amplitude reflectivity profiles, diffractive optical elements, and various kinds of adaptive resonator mirrors. Major contributions to research in this area have come from groups led by Belanger [38], [214]–[217] and Leger [218]–[223]. Continued advances in this area of customized resonators and of passive-mode conjugation can be expected.

B. Diode Laser Resonators

Semiconductor diode lasers, including both conventional horizontal stripe lasers and the newer vertical-cavity surface-emit-

ting (VCSEL) lasers, represent a dominant portion of current laser research and also of the future market potential for lasers. Many of the resonator concepts described in this paper are also relevant to diode lasers (although this may not always be recognized by diode laser workers having only semiconductor fabrication backgrounds).

Resonator designs for diode lasers are constrained by the almost invisibly small size of diode lasers and by the need in most cases to fabricate the resonator structure directly on or within the diode laser itself. Great progress has been made in developing diode laser resonators under these constraints but many interesting challenges and possibilities remain.

C. Laser Beam Quality

The skeptical observer might note that over the past four decades the modal properties of practical laser devices have been much more frequently calculated than measured. The meaningful measurement of laser beam properties has, however, made substantial progress during the past decade with the emergence of the rigorously defined second-moment-based " M -squared" parameter [224], together with many new instruments to measure laser beam properties.

The M^2 value in particular, although a useful parameter for characterizing a laser beam, is best viewed as providing only a "propagation factor" for a laser beam, rather than a unique or universal measure of laser beam quality, since the "quality" of a laser beam depends on the application for which it is intended, and this is not in every case directly related to the M^2 value. The accurate measurement of M^2 and other beam spatial properties can nonetheless provide much useful information on the merits of a given laser beam for practical applications, and also on the modal characteristics and the basic laser physics of laser devices. Continued attention to methods for characterizing real laser beams, along with increased emphasis on really measuring the modal characteristics of real laser devices, should be encouraged.

D. Ultra Low-Loss Resonators

Finally, advances in mirror coating technology in recent years have made possible near-perfect optical mirrors with power reflections in the "five nines" range, leading to optical resonators with parts per million losses per round trip. This in turn has made possible new advances in ring laser gyroscopes, laser gravity-wave interferometers, cavity ring-down spectroscopy, laser electron accelerators, and sophisticated experiments that explore the basic quantum properties of atoms in extraordinarily high- Q optical cavities. Advances in this area will obviously become more difficult as the technologies near perfection, but the possibilities of new fundamental results are also similarly enhanced.

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On behalf of many other colleagues, I would also like to express great regrets over the death on November 28, 2000, of Yurii Alekseevich Anan'ev, for many years the leading Russian contributor to the field covered by these reviews.

REFERENCES

- [1] A. E. Siegman, "Laser beams and resonators: The 1960s," *IEEE J. Select. Topics Quantum Electron.*, vol. 6, 2000.
- [2] G. A. Deschamps and P. E. Mast, "Beam tracing and applications," in *Quasi Optics*, J. Fox, Ed. New York: Polytechnic Institute of Brooklyn Press, 1964, pp. 379–395.
- [3] J. B. Keller and W. Streifer, "Complex rays with an application to Gaussian beams," *J. Opt. Soc. Amer.*, vol. 61, pp. 40–43, Jan. 1971.
- [4] G. A. Deschamps, "Ray techniques in electromagnetics," *Proc. IEEE*, vol. 60, pp. 1022–1035, Sept. 1972.
- [5] J. A. Arnaud, "Hamiltonian theory of beam mode propagation," in *Progress in Optics*, E. Wolf, Ed. Amsterdam, The Netherlands: North-Holland, 1973, vol. 11, pp. 247–304.
- [6] R. Simon, E. C. G. Sudarshan, and N. Mukunda, "Generalized rays in first-order optics: Transformation properties of Gaussian Schell-model fields," *Phys. Rev. A*, vol. 29, pp. 3273–3279, 1984.
- [7] H. Kogelnik, "On the propagation of Gaussian beams of light through lenslike media including those with a loss or gain variation," *Appl. Opt.*, vol. 4, pp. 1562–1569, Dec. 1965.
- [8] L. W. Casperson and A. Yariv, "The Gaussian mode in optical resonators with a radial gain profile," *Appl. Phys. Lett.*, vol. 12, pp. 355–357, May 15, 1968.
- [9] H. Zucker, "Optical resonators with variable reflectivity mirrors," *Bell Sys. Tech. J.*, vol. 49, pp. 2349–2376, Nov. 1970.
- [10] A. E. Siegman, "Hermite–Gaussian functions of complex argument as optical beam eigenfunctions," *J. Opt. Soc. Amer.*, vol. 63, pp. 1093–1094, Sept. 1973.
- [11] A. E. Siegman, P. L. Mussche, and J.-I. Doumont, "Excess spontaneous emission factor in variable-reflectivity-mirror lasers," *IEEE J. Quantum Electron.*, vol. QE-25, pp. 1960–1967, Aug. 1989.
- [12] J. Alda, S. Wang, and E. Bernabeu, "Analytical expression for the complex radius of curvature tensor Q for generalized Gaussian beams," *Opt. Commun.*, vol. 80, pp. 350–353, Jan. 15, 1991.
- [13] P. A. Belanger, "Beam propagation and the ABCD ray matrices," *Opt. Lett.*, vol. 16, pp. 196–198, Feb. 15, 1991.
- [14] S. E. Miller, "Light propagation in generalized lens-like media," *Bell Sys. Tech. J.*, vol. 44, pp. 2017–2064, Nov. 1964.
- [15] P. K. Tien, J. P. Gordon, and J. R. Whinnery, "Focusing of a light beam of Gaussian field distribution in continuous and periodic lens-like media," *Proc. IEEE*, vol. 53, pp. 129–136, Feb. 1965.
- [16] C. N. Kurtz and W. Streifer, "Guided waves in inhomogeneous focusing media I," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 11–15, Jan. 1969.
- [17] —, "Guided waves in inhomogeneous focusing media II," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 250–253, May 1969.
- [18] A. Hardy, "Beam propagation through parabolic-index waveguides with distorted optical axis," *Appl. Phys.*, vol. 18, pp. 223–226, 1979.
- [19] L. W. Casperson, "Mode stability of lasers and periodic optical systems," *IEEE J. Quantum Electron.*, vol. QE-10, pp. 629–634, Sept. 1974.
- [20] L. W. Casperson and S. D. Lunnam, "Gaussian modes in high loss laser resonators," *Appl. Opt.*, vol. 14, pp. 1193–1199, May 1975.
- [21] U. Ganiel, A. Hardy, and Y. Silberberg, "Stability of optical laser resonators with mirrors of Gaussian reflectivity profile, which contain an active medium," *Opt. Commun.*, vol. 14, pp. 290–293, July 1975.
- [22] A. Yariv and P. Yeh, "Confinement and stability in optical resonators employing mirrors with Gaussian reflectivity tapers," *Opt. Commun.*, vol. 13, pp. 370–374, Apr. 1975.
- [23] L. W. Casperson, "Beam modes in complex lenslike media and resonators," *J. Opt. Soc. Amer.*, vol. 66, pp. 1373–1379, Dec. 1976.
- [24] T. L. Paoli, "Waveguiding in a stripe-geometry junction laser," *IEEE J. Quantum Electron.*, vol. QE-13, pp. 662–668, Aug. 1977.
- [25] R. Prates and L. Ronchi, "Generalized Gaussian beams in free space," *J. Opt. Soc. Amer.*, vol. 67, pp. 1274–1276, Sept. 1977.
- [26] M. Nazarathy, A. Hardy, and J. Shamir, "Generalized mode propagation in first-order optical systems with loss or gain," *J. Opt. Soc. Amer.*, vol. 72, pp. 1409–1420, Oct. 1982.
- [27] —, "Generalized mode theory of conventional and phase-conjugate resonators," *J. Opt. Soc. Amer.*, vol. 73, pp. 576–586, May 1983.
- [28] B. N. Perry, P. Rabinowitz, and M. Newstein, "Wave propagation in media with focused gain," *Phys. Rev. A*, vol. 27, pp. 1989–2002, Apr. 1983.
- [29] Y. A. Anan'ev and S. G. Anikichev, "Eigenfunction series expansions of open cavity resonators," *Opt. Spectros. (USSR)*, vol. 61, pp. 537–539, Oct. 1986.
- [30] Y. A. Anan'ev and A. Y. Bekshaev, "Matrix analysis of optical systems with arbitrary quadratic phase-amplitude correctors," *Opt. Spectros. (USSR)*, vol. 61, pp. 702–705, Nov. 1986.
- [31] S. Lavi, R. Prochaska, and E. Keren, "Generalized beam parameters and transformation laws for partially coherent light," *Appl. Opt.*, vol. 27, pp. 3696–3703, Sept. 1, 1988.
- [32] R. Simon, N. Mukunda, and E. C. G. Sudarshan, "Partially coherent beams and a generalized ABCD-law," *Opt. Commun.*, vol. 65, pp. 322–328, Mar. 1, 1988.
- [33] M. J. Bastiaans, "Propagation laws for the second-order moments of the Wigner distribution function in first-order optical systems," *Optik*, vol. 82, pp. 173–181, 1989.
- [34] J.-I. Doumont, "Laser beam and resonator calculations on desktop computers," Ph.D. dissertation, Dept. of Applied Physics, Stanford Univ., Stanford, CA, June 1991.
- [35] C. Pare and P. A. Belanger, "Beam propagation in a linear or nonlinear medium with a parabolic index profile using ABCD ray matrices: The method of moments," *Opt. Quantum Electron.*, vol. 24, pp. S1051–S1070, Sept. 1992.
- [36] M. A. Porras, J. Alda, and E. Bernabeu, "Complex beam parameter and ABCD law for non-Gaussian and nonspherical light beams," *Appl. Opt.*, vol. 31, pp. 6389–6402, 1992.
- [37] F. Salin and J. Squier, "Gain guiding in solid-state lasers," *Opt. Lett.*, vol. 17, pp. 1352–1354, Oct. 1, 1992.
- [38] P.-A. Belanger and C. Pare, "Unstable laser resonators with a specified output profile by using a graded-reflectivity mirror: Geometrical optics limit," *Optics Commun.*, vol. 109, pp. 507–517, July 15, 1994.
- [39] A.-A. R. Al-Rashed and B. E. A. Saleh, "Decentered Gaussian beams," *Appl. Opt.*, vol. 34, pp. 6819–6825, Oct. 20, 1995.
- [40] Y. Sun, "Lateral mode control of semiconductor lasers," Ph.D. dissertation, Dept. of Electrical Engineering, Stanford Univ., Stanford, CA, Aug. 1995.
- [41] Y. Sun and A. E. Siegman, "Optical mode properties of laterally offset gain and index guiding structures," *IEEE J. Quantum Electron.*, vol. 32, pp. 790–795, May 1996.
- [42] A. Kostenbauder, Y. Sun, and A. E. Siegman, "Eigenmode expansions using biorthogonal eigenfunctions: Complex-valued Hermite Gaussians," *J. Opt. Soc. Amer. A*, vol. 14, pp. 1780–1790, Aug. 1997.
- [43] A. E. Siegman, "Advances in laser resonator design using variable reflectivity mirrors," in *Tutorials in Optics*, D. T. Moore, Ed. Washington, D.C.: Optical Society of America, 1992, pp. 65–84.
- [44] P. Sona, P. Muys, C. Leys, and G. Sherman, "Variable reflectivity output coupler for improvement of the beam quality of a fast axial flow CO₂-laser," in *Gas Flow and Chemical Lasers*, Proc. SPIE, vol. 1397, Sept. 1990.
- [45] A. Caprara and G. C. Reali, "Time varying M^2 in Q -switched lasers," *Opt. Quantum Electron.*, vol. 24, pp. S1001–S1009, Sept. 1992.
- [46] —, "Time-resolved M^2 of nanosecond pulses from a Q -switched variable-reflectivity-mirror Nd:YAG laser," *Opt. Lett.*, vol. 17, pp. 414–416, Mar. 15, 1992.
- [47] G. Duplain, P. G. Verly, A. J. Dobrowolski, A. Waldorf, and S. Bussiere, "Graded-reflectance mirrors for beam quality control in laser resonators," *Appl. Opt.*, vol. 32, pp. 1145–1153, Mar. 1, 1993.
- [48] G. Cerullo, S. De Silvestri, V. Magni, and O. Svelto, "Resonator designs for high-brightness solid-state lasers," in *Solid State Lasers: New Developments and Applications*, M. Inguscio and R. Wallenstein, Eds. New York: Plenum Press, 1994, pp. 1–12.
- [49] S. E. Kovalenko, V. Losev, and M. R. Perrone, "Super-Gaussian resonators for long-pulse XeCl lasers," *Appl. Opt.*, vol. 33, pp. 4082–4086, June 20, 1994.

- [50] G. Bostanjoglo and H. Weber, "Improvement of power-laser characteristics with optimized gradient reflectivity mirrors," *Laser Optoelektron.*, vol. 4, pp. 51–61, Aug. 1996.
- [51] D. Stryckman, G. Rousseau, M. D' Auteuil, and N. McCarthy, "Improvement of the lateral-mode discrimination of broad-area diode lasers with a profiled reflectivity output facet," *Appl. Opt.*, vol. 35, pp. 5955–5959, Oct. 20, 1996.
- [52] J. A. Arnaud, "Modes of propagation of optical beams in helical gas lenses," *Proc. IEEE*, vol. 59, pp. 1378–1379, Sept. 1971.
- [53] A. T. Friberg, E. Tervonen, and J. Turunen, "Twisted Gaussian-Schell-model beams: Interpretation and experiment," in *Optics as a Key to High Technology*, Proc. SPIE, vol. 1983, 1993.
- [54] R. Simon and N. Mukunda, "Twisted Gaussian Schell model beams," *J. Opt. Soc. Amer. A*, vol. 10, pp. 95–109, 1993.
- [55] M. W. Beijersbergen *et al.*, "Helical-wavefront laser beams produced with a spiral phaseplate," *Opt. Commun.*, vol. 112, pp. 321–327, 1994.
- [56] A. T. Friberg, E. Tervonen, and J. Turunen, "Interpretation and experimental demonstration of twisted Gaussian-Schell-model beams," *J. Opt. Soc. Amer. A*, vol. 11, 1994.
- [57] G. Sleky, C. O. Weiss, D. Y. Tang, and M. F. H. Tarroja, "Helical-wave emission of lasers," *J. Opt. Soc. Amer. B*, vol. 11, pp. 2089–2094, Oct. 1994.
- [58] J. A. Arnaud and H. Kogelnik, "Gaussian light beams with general astigmatism," *Appl. Opt.*, vol. 8, pp. 1687–1693, Aug. 1969.
- [59] J. A. Arnaud, "Degenerate optical cavities. II. Effects of misalignments," *Appl. Opt.*, vol. 8, pp. 1909–1917, Sept. 1969.
- [60] —, "Degenerate optical cavities," *Appl. Opt.*, vol. 8, pp. 189–195, Jan. 1969.
- [61] —, "Nonorthogonal optical waveguides and resonators," *Bell Sys. Tech. J.*, vol. 49, pp. 2311–2348, Nov. 1970.
- [62] —, "Optical resonators in the approximation of Gauss," *Proc. IEEE*, vol. 62, pp. 1561–1570, Nov. 1974.
- [63] —, *Beam and Fiber Optics*. New York: Academic, 1976.
- [64] G. Nemes, I. E. Teodorescu, and M. Nemes, "On the phase-space description of optical and particle beams," in *Proc. 3rd Int. Sch. Coherent Optics*, I. Ursu and A. M. Prokhorov, Eds, 1983.
- [65] G. Nemes, "Measuring and handling general astigmatic beams," in *Laser Beam Characterization*, P. M. Mejias, H. Weber, R. Martinez-Herrero, and A. Gonzalez-Urena, Eds. Madrid, Spain: SEDO (Optical Society of Spain), 1993, pp. 325–358.
- [66] G. Nemes and A. G. Kostenbauder, "Optical systems for rotating a beam," in *Laser Beam Characterization*, P. M. Mejias, H. Weber, R. Martinez-Herrero, and A. Gonzalez-Urena, Eds. Madrid, Spain: SEDO (Optical Society of Spain), 1993, pp. 99–109.
- [67] G. Nemes and A. E. Siegman, "Measurement of all ten second moments of an astigmatic beam by the use of rotating simple astigmatic (anamorphic) optics," *J. Opt. Soc. Amer. A*, vol. 11, pp. 2257–2264, Aug. 1994.
- [68] G. Nemes, "Synthesis of general astigmatic optical systems, the detwisting procedure, and the beam quality factors of general astigmatic beams," in *Laser Beam Characterization*, H. Weber, N. Reng, L. Ludtke, and P. M. Mejias, Eds. Berlin, Germany: Festkorper-Laser-Institute Berlin, GmbH, May–June 1994, pp. 93–104.
- [69] G. Nemes and J. Serna, "The ten physical parameters associated with a full general astigmatic beam: A Gauss-Schell model," in *Laser Beam and Optics Characterization (LBOC 4)*, A. Giesen and M. Morin, Eds. Munich, Germany: VDI Technologiezentrum, June 1997, pp. 92–105.
- [70] —, "Laser beam characterization with use of second order moments: An overview (Invited talk)," in *DPSS Lasers: Applications and Issues (OSA TOPS Vol. 17)*, M. W. Dowley, Ed. Washington, D.C.: Optical Society of America, 1998, pp. 200–207.
- [71] L. W. Casperson, "Synthesis of Gaussian-beam optical systems," *Appl. Opt.*, vol. 20, pp. 2243–2249, July 1, 1981.
- [72] R. Martinez-Herrero, P. M. Mejias, and C. Martinez, "Parametric characterization of the phase at the far field," *Opt. Lett.*, vol. 20, pp. 651–653, Apr. 1, 1995.
- [73] Y. Lu, "Mode analysis of axially-symmetric optical resonators," *Chin. J. Lasers*, vol. 1, pp. 117–123, Apr. 1992.
- [74] B. Lu, S. Xu, Y. Hu, and B. Cai, "Matrix representation of three-dimensional astigmatic resonators," *Opt. Quantum Electron.*, vol. 24, pp. 619–630, 1992.
- [75] L. Qiang, W. Shaomin, J. Alda, and E. Bernabeu, "Transformation of nonsymmetric Gaussian beam into symmetric one by means of tensor ABCD law," *Optik*, vol. 85, pp. 67–72, 1990.
- [76] B. T. Landesman, "Geometrical representation of the fundamental mode of a Gaussian beam in oblate spheroidal coordinates," *J. Opt. Soc. Amer. A*, vol. 6, pp. 5–17, Jan. 1989.
- [77] J. Serna, R. Martinez-Herrero, and P. M. Mejias, "Parametric characterization of general partially coherent beams propagating through ABCD optical systems," *J. Opt. Soc. Amer. A*, vol. 8, pp. 1094–1098, July 1991.
- [78] J. Serna, P. M. Mejias, and R. Martinez-Herrero, "Rotation of partially coherent beams propagating through free space," *Opt. Quantum Electron.*, vol. 24, pp. S873–S880, Sept. 1992.
- [79] G. B. Alshuler, N. R. Belashenkov, V. B. Karasev, V. Y. Khramov, and L. V. Khloponin, "Resonators with field rotation," in *Laser Resonators and Coherent Optics: Modeling, Technology, and Applications*, Proc. SPIE, vol. 1868, Jan. 1993.
- [80] V. Bagini, F. Gori, M. Santarsiero, F. Freeza, G. Schettini, M. Richetta, and G. S. Spagnolo, "On a class of twisting beams," in *Laser Beam Characterization*, P. M. Mejias, H. Weber, R. Martinez-Herrero, and A. Gonzalez-Urena, Eds. Madrid, Spain: SEDO (Optical Society of Spain), June 14–16, 1993, pp. 31–40.
- [81] Kauderer, "Modes in n -dimensional first-order systems," *J. Math. Phys.*, vol. 34, pp. 4221–4250, Sept. 1993.
- [82] B. Lu, G. Feng, and B. Cai, "Complex ray representation of the astigmatic Gaussian beam propagation," *Opt. Quantum Electron.*, vol. 25, pp. 275–284, 1993.
- [83] J. Serna and G. Nemes, "Decoupling of coherent Gaussian beams with general astigmatism," *Opt. Lett.*, vol. 18, pp. 1774–1776, Nov. 1, 1993.
- [84] A. T. Friberg, E. Tervonen, and J. Turunen, "Interpretation and experimental demonstration of twisted Gaussian Schell-model beams," *J. Opt. Soc. Amer. A*, vol. 11, 1994, to be published.
- [85] —, "Focusing of twisted Gaussian-Schell-model beams," *Opt. Commun.*, vol. 106, pp. 127–132, 1994.
- [86] A. T. Friberg, "Optical twist," in *International Trends in Optics and Photonics, ICO*, T. Asakura, Ed. New York: Springer-Verlag, 1999, vol. IV.
- [87] E. A. Sziklas and A. E. Siegman, "Mode calculations in unstable resonators with flowing saturable gain. I. Hermite-Gaussian expansion," *Appl. Opt.*, vol. 13, pp. 2775–2792, Dec. 1974.
- [88] P. M. Duffieux, *L'Integrale de Fourier et ses Applications a L'Optique*: Faculte des Sciences, 1946.
- [89] J. A. Ratcliffe, "Some aspects of diffraction theory and their application to the ionosphere," in *Rep. Prog. Phys.*, A. C. Strickland, Ed. London, U.K.: The Physical Society, 1956, vol. 19, p. 188.
- [90] E. A. Sziklas and A. E. Siegman, "Diffraction calculations using fast Fourier transform methods," *Proc. IEEE*, vol. 62, pp. 410–412, Mar. 1974.
- [91] —, "Mode calculations in unstable resonator with flowing saturable gain. II. Fast Fourier transform method," *Appl. Opt.*, vol. 14, pp. 1873–1889, Aug. 1975.
- [92] M. Johnson, "Direct application of the fast Fourier transform to open resonator calculations," *Appl. Opt.*, vol. 13, pp. 2326–2328, Oct. 1974.
- [93] G. N. Lawrence, "Optical system analysis with physical optics codes," in *Proc. SPIE*, vol. 766, 1987.
- [94] —, "Optical design with physical optics using GLAD," in *Int. Lens Design Conf.*, Proc. SPIE, vol. 1354, 1990.
- [95] G. N. Lawrence and S. H. Hwang, "Beam propagation in gradient refractive-index media," *Appl. Opt.*, vol. 31, pp. 5201–5210, 1992.
- [96] A. E. Siegman, "Quasi fast Hankel transform," *Opt. Lett.*, vol. 1, pp. 13–15, Mar. 1977.
- [97] S.-C. Sheng, "Studies of laser resonators and beam propagation using fast transform methods," Ph.D dissertation, Dept. of Applied Physics, Stanford Univ., Stanford, CA, Mar. 1980.
- [98] S.-C. Sheng and A. E. Siegman, "Nonlinear optical calculations using fast transform methods: Second harmonic generation with depletion and diffraction," *Phys. Rev. A*, vol. 21, pp. 599–606, Feb. 1980.
- [99] A. G. Fox and T. Li, "Computation of optical resonator modes by the method of resonance excitation," *IEEE J. Quantum Electron.*, vol. QE-4, pp. 460–465, July 1968.
- [100] R. L. Sanderson and W. Streifer, "Comparison of laser mode calculations," *Appl. Opt.*, vol. 8, pp. 131–136, July 1969.
- [101] G. C. F. M. Riche and B. de Prony, "Essai experimental et analytique (sur les lois de la dilatibilite des fluides elastiques et sur celles de la force expansive de la vapeur de l'eau et de la vapeur de l'alkool a differentes temperatures)," *J. l'Ecole Polytech. (Paris)*, vol. t. 1, 2eme cahier (An. III), pp. 26–27, Dec. 1795.
- [102] D. F. Tuttle, "Network synthesis for prescribed transient response," Ph.D dissertation, Dept. of Electrical Engineering, Massachusetts Inst. of Technology, Cambridge, MA, 1948.
- [103] A. E. Siegman and H. Y. Miller, "Unstable optical resonator loss calculations using the Prony method," *Appl. Opt.*, vol. 9, pp. 2729–2763, Dec. 1970.

- [104] W. D. Murphy and M. L. Bernabe, "Numerical procedures for solving nonsymmetric eigenvalue problems associated with optical resonators," *Appl. Opt.*, vol. 17, pp. 2358–2365, Aug. 1, 1978.
- [105] G. C. Dente, "Polarization effects in resonators," *Appl. Opt.*, vol. 18, pp. 2911–2912, Sept. 1, 1979.
- [106] E. Gerck, "Prony method revisited," *Appl. Opt.*, vol. 18, p. 3075, Sept. 15, 1979.
- [107] W. P. Latham Jr. and G. C. Dente, "Matrix methods for bare resonator eigenvalue analysis," *Appl. Opt.*, vol. 19, pp. 1618–1621, May 15, 1980.
- [108] S. V. Rulev and V. A. Shcheglov, "Prony-method calculation of the mode structure of the HSURIA resonator," *Kvantovaya Elektron., Mosk.*, vol. 20, pp. 611–612, June 1993.
- [109] J. Klein, "Optimierte Grundmode-Laserstrahlungsquellen," Ph.D. dissertation, Fraunhofer Institut für Lasertechnik (Verlag Shaker), 1994.
- [110] T. F. Johnston Jr., M. W. Sasnett, J.-I. Doumont, and A. E. Siegman, "Laser beam quality versus aperture size in a CW argon-ion laser," *Opt. Lett.*, vol. 17, pp. 198–200, Feb. 1, 1992.
- [111] J. P. Gordon, "Optics of general guiding media," *Bell Sys. Tech. J.*, vol. 45, pp. 321–332, 1966.
- [112] D. Gloge and D. Marcuse, "Formal quantum theory of light rays," *J. Opt. Soc. Amer.*, vol. 59, pp. 1629–1631, Dec. 1969.
- [113] W. E. Lamb Jr., "Nonrelativistic quantum mechanics," *Phys. Today*, vol. 22, pp. 23–28, Apr. 1969.
- [114] M. Feldman, "Modes in general Gaussian optical systems using quantum-mechanics formalism," *J. Opt. Soc. Amer.*, vol. 61, pp. 446–449, Apr. 1971.
- [115] E. E. Bergmann, "Optical resonators with paraxial modes," *Appl. Opt.*, vol. 11, pp. 113–119, Jan. 1972.
- [116] D. Stoler, "Operator methods in physical optics," *J. Opt. Soc. Amer.*, vol. 71, pp. 334–341, Mar. 1981.
- [117] M. Nazarathy, "Operator methods in first order optics," Ph.D. dissertation (in Hebrew), Israel Inst. of Technology, 1982.
- [118] M. Nazarathy and J. Shamir, "First order optics—A canonical operator representation, lossless systems," *J. Opt. Soc. Amer.*, vol. 72, p. 356, 1982.
- [119] —, "First order optics: Operator representation for systems with loss or gain," *J. Opt. Soc. Amer.*, vol. 72, p. 1398, 1982.
- [120] C. Pare, L. Gagnon, and P. A. Belanger, "Aspherical laser resonators: An analogy with quantum mechanics," *Phys. Rev. A*, vol. 46, pp. 4150–4160, Oct. 1, 1992.
- [121] K. Wittig, H. Hugel, and A. Giesen, "Paraxial optics in terms of quantum mechanics," in *Laser Beam Characterization*, P. M. Mejias, H. Weber, R. Martinez-Herrero, and A. Gonzalez-Urena, Eds. Madrid, Spain: SEDO (Optical Society of Spain), June 14–16, 1993, pp. 185–196.
- [122] K. Wittig, A. Giesen, and H. Hugel, "An algebraic approach to characterize paraxial optical systems," *Appl. Opt.*, vol. 33, pp. 3837–3848, 1994.
- [123] I. F. Balashov and V. A. Berenberg, "Nonstationary modes of an open resonator," *Sov. J. Quantum Electron.*, vol. 5, pp. 151–161, Aug. 1975.
- [124] A. E. Siegman, "Orthogonality properties of optical resonator eigenmodes," *Opt. Commun.*, vol. 31, pp. 369–373, Dec. 1979.
- [125] A. E. Siegman, "Exact cavity equations for lasers with large output coupling," *Appl. Phys. Lett.*, vol. 36, pp. 412–414, Mar. 15, 1980.
- [126] E. M. Wright and W. J. Firth, "Orthogonality properties of general optical resonator eigenmodes," *Opt. Commun.*, vol. 40, pp. 410–412, Feb. 15, 1982.
- [127] E. M. Wright, D. P. O'Brien, and W. J. Firth, "Reciprocity and orthogonality relations for ring resonators," *IEEE J. Quantum Electron.*, vol. QE-20, pp. 1307–1310, 1984.
- [128] M. Piche and P. Belanger, "On the losses of the counterpropagating modes of ring cavities," *IEEE J. Quantum Electron.*, vol. QE-20, pp. 1303–1307, 1984.
- [129] J. A. Benda, G. E. Palma, and E. A. Sziklas, "Theory of ring lasers coupled using reverse waves," in *Modeling and Simulation of Laser Systems*, Proc. SPIE, vol. 1045, 1989.
- [130] D. L. Bullock and J. S. Yun, "Fundamental beam quality considerations for modes of adjoined-coupled lasers," in *Modeling and Simulation of Laser Systems*, Proc. SPIE, vol. 1045, 1989.
- [131] G. H. C. New, "The origin of excess noise," *J. Mod. Opt.*, vol. 42, pp. 799–810, Apr. 1995.
- [132] A. E. Siegman, "Lasers without photons—Or should it be lasers with too many photons?," *Appl. Phys. B*, vol. 60, pp. 247–257, Feb./Mar. 1995.
- [133] —, "Lasers without photons," in *Coherence and Quantum Optics VII*, J. H. Eberly, L. Mandel, and E. Wolf, Eds. New York: Plenum Press, June 1995, pp. 229–238.
- [134] M. Sargent III, M. O. Scully, and W. E. Lamb Jr., *Laser Physics*. Reading, MA: Addison-Wesley, 1972.
- [135] S. P. Morgan, "On the integral equations of laser theory," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 191–193, May 1963.
- [136] K. Petermann, "Calculated spontaneous emission factor for double-heterostructure injection lasers with gain-induced waveguiding," *IEEE J. Quantum Electron.*, vol. QE-15, pp. 566–570, July 1979.
- [137] A. E. Siegman, "Excess spontaneous emission in nonhermitian optical systems—I: Laser amplifiers," *Phys. Rev. A*, vol. 39, pp. 1253–1263, Feb. 1, 1989.
- [138] —, "Excess spontaneous emission in nonhermitian optical systems. II. Laser oscillators," *Phys. Rev. A*, vol. 39, pp. 1264–1268, Feb. 1, 1989.
- [139] P. Grangier and J.-P. Poizat, "A simple quantum picture for the Petermann excess noise factor," *Eur. Phys. J. D*, vol. 1, pp. 97–104, Jan. 1998.
- [140] Y.-J. Cheng, P. L. Mussche, and A. E. Siegman, "Measurement of laser quantum frequency fluctuations using a Pound–Drever stabilization system," *IEEE J. Quantum Electron.*, vol. QE-30, pp. 1498–1504, June 1994.
- [141] P. Mussche, "Excess quantum linewidth in lasers with nonorthogonal eigenmodes," Ph.D. dissertation, Dept. of Electrical Engineering, Stanford Univ., Stanford, CA, Dec. 1994.
- [142] Y.-J. Cheng, G. Fanning, and A. E. Siegman, "Experimental observation of a large excess quantum noise factor in the linewidth of a laser oscillator using nonorthogonal modes," *Phys. Rev. Lett.*, vol. 77, pp. 627–630, July 22, 1996.
- [143] Y.-J. Cheng, "Excess quantum noise in laser oscillators using nonorthogonal modes," Ph.D. dissertation, Dept. of Applied Physics, Stanford Univ., Stanford, CA, Dec. 1996.
- [144] M. A. van Eijkelenborg, A. M. Lindberg, M. S. Thijssen, and J. P. Woerdman, "Resonance of quantum noise in an unstable cavity laser," *Phys. Rev. Lett.*, vol. 77, pp. 4314–4317, Nov. 18, 1996.
- [145] A. N. van der Lee, N. J. van Druten, A. Mieremet, M. A. van Eijkelenborg, A. M. Lindberg, M. P. van Exter, and J. P. Woerdman, "Excess quantum noise due to nonorthogonal polarization modes," *Phys. Rev. Lett.*, vol. 79, pp. 4357–4360, 1997.
- [146] M. A. van Eijkelenborg, "Excess quantum noise in unstable-cavity lasers," Ph.D. dissertation, Huygens' Institute, Rijksuniversiteit te Leiden, The Netherlands, 1997.
- [147] O. Emile, M. Brunel, F. Bretenaker, and A. Le Floch, "Direct measurement of the excess noise factor in a geometrically stable laser resonator," *Phys. Rev.*, vol. 57, pp. 4889–4893, June 1998.
- [148] O. Emile, M. Brunel, A. Le Floch, and F. Bretenaker, "Vectorial excess noise factor in common lasers," *Europhys. Lett.*, vol. 43, pp. 153–157, July 15, 1998.
- [149] G. P. Karman, A. M. Lindberg, and J. P. Woerdman, "Observed factorization of excess quantum noise that is due to both polarization and spatial mode nonorthogonality," *Opt. Lett.*, vol. 23, pp. 1698–1700, Nov. 1, 1998.
- [150] M. V. Berry, "The geometric phase," *Sci. Amer.*, vol. 259, no. 6, pp. 26–32, Dec. 1988.
- [151] Y. K. Danileiko and V. A. Lobachev, "New rotating-field resonator for lasers," *Sov. J. Quantum Electron.*, vol. 1, pp. 389–390, Sept. 1974.
- [152] —, "New resonator for optical oscillator—Resonator with field rotation," *Kvantovaya Elektron., Mosk.*, vol. 1, pp. 668–671, 1974.
- [153] M. M. Popov, "Resonators for lasers with rotated directions of principal curvatures," *Opt. Spectros.*, vol. 25, pp. 170–171, 1968.
- [154] —, "Resonators for lasers with unfolded directions of principal curvatures," *Opt. Spectros.*, vol. 25, pp. 213–217, 1968.
- [155] T. F. Pankratova, "Natural frequencies and polarizations of a ring resonator," *Opt. Spektros.*, vol. 36, pp. 969–974, May 1974.
- [156] L. A. Belousova, "Theory of nonorthogonal resonators," *J. Appl. Spectros. (USSR)*, vol. 30, pp. 172–175, 1979.
- [157] F. Biraben, "Efficacite des systemes unidirectionnels utilisables dans les lasers en anneau," *Opt. Commun.*, vol. 29, pp. 353–356, June 1979.
- [158] S. I. Zavgorodnava, V. I. Kuprenyuk, and V. E. Sherstobitov, "Unstable resonator with field rotation," *Sov. J. Quantum Electron.*, vol. 7, pp. 787–788, June 1977.
- [159] Y. A. Anan'ev, "Unstable laser resonator for low-gain media," *Sov. Tech. Phys. Lett.*, vol. 4, pp. 150–151, 1978.
- [160] Y. A. Anan'ev, V. I. Kuprenyuk, and V. E. Sherstobitov, "Properties of unstable resonators with field rotation. I. Theoretical principles," *Sov. J. Quantum Electron.*, vol. 9, pp. 1105–1110, Sept. 1979.
- [161] D. A. Goryachkin, V. P. Kalinin, V. I. Kuprenyuk, V. V. Sergeev, and V. E. Sherstobitov, "Properties of unstable resonators with field rotation. II. Experimental results," *Sov. J. Quantum Electron.*, vol. 9, pp. 1110–1114, Sept. 1979.
- [162] V. I. Kuprenyuk and V. E. Sherstobitov, "Calculations on the mirror system of an unstable resonator with field rotation," *Sov. J. Quantum Electron.*, vol. 10, pp. 449–453, Apr. 1980.

- [163] V. I. Kuprenyuk, V. E. Semenov, L. D. Smirnova, and V. E. Sherstobitov, "Wave-approximation calculation of an unstable resonator with field rotation," *Sov. J. Quantum Electron.*, vol. 13, pp. 1613–1617, 1983.
- [164] A. H. Paxton and W. P. Latham Jr., "Ray matrix method for the analysis of optical resonators with image rotation," in *Int. Lens Design Conf., Proc. SPIE*, vol. 554, June 1985.
- [165] —, "Unstable resonators with 90 degree beam rotation," *Appl. Opt.*, vol. 25, pp. 2939–2945, 1986.
- [166] A. H. Paxton, "Unstable ring resonator with an intracavity prism beam expander," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 241–244, Feb. 1987.
- [167] S. Holswade, R. Riviere, C. A. Huguley, C. M. Clayton, and G. C. Dente, "Experimental evaluation of an unstable ring resonator with 90 degrees beam rotation: HiQ experimental results," *Appl. Opt.*, vol. 27, pp. 4396–4406, Nov. 1, 1988.
- [168] W. P. Latham Jr., A. H. Paxton, and G. C. Dente, "Laser with 90-degree beam rotation," in *Optical Resonators, Proc. SPIE*, vol. 1224, Jan. 1990.
- [169] G. B. Jacobs, "CO₂ laser gyro using polarizationally isotropic cavity," *Appl. Opt.*, vol. 10, pp. 220–221, Jan. 1971.
- [170] I. W. Smith and T. Dorschner, "Electromagnetic wave ring resonator," U.S. Patent 4 110 045, Jan. 2, 1976.
- [171] V. E. Sanders and D. Z. Anderson, "Isotropic nonplanar ring resonator," U.S. Patent 4 247 832, Jan. 27, 1981.
- [172] T. A. Dorschner, "Nonplanar rings for laser gyroscopes," in *Fiber Optic and Laser Sensors, Proc. SPIE*, vol. 412, Apr. 1983.
- [173] H. Statz, T. A. Dorschner, M. Holtz, and I. W. Smith, "The multioscillator ring laser gyroscope," in *Laser Handbook*, M. L. Stutch and M. Bass, Eds. Amsterdam, The Netherlands: North-Holland, 1985, vol. 4, ch. 3.
- [174] Y. D. Golyaev, K. N. Evtukhov, and L. N. Kaptsov, "Spatial and polarization characteristics of radiation from a CW neodymium-doped garnet laser with a nonplanar ring resonator," *Sov. J. Quantum Electron.*, vol. 11, pp. 1421–1426, Nov. 1981.
- [175] —, "Temporal and spectral characteristics of radiation from a CW neodymium-doped garnet laser with a nonplanar ring resonator," *Sov. J. Quantum Electron.*, vol. 11, pp. 1427–1432, Nov. 1981.
- [176] T. J. Kane and R. L. Byer, "Monolithic, unidirectional single-mode Nd:YAG ring laser," *Opt. Lett.*, vol. 10, pp. 65–67, Feb. 1985.
- [177] T. J. Kane and R. L. Byer, "Solid-state nonplanar internally reflecting ring laser," U.S. Patent 4 578 793, 1986.
- [178] G. B. Altschuler, E. D. Isyanova, V. B. Karasev, A. L. Levit, V. M. Ovchinnikov, and S. F. Sharlai, "Analysis of misalignment sensitivity of ring laser resonators," *Sov. J. Quantum Electron.*, vol. 7, pp. 857–859, 1977.
- [179] I. W. Smith, "Optical resonator axis stability and instability from first principles," in *Fiber Optic and Laser Sensors, Proc. SPIE*, vol. 412, Apr. 1983.
- [180] A. L. Levit and V. M. Ovchinnikov, "Stability of a ring resonator with a nonplane axial contour," *J. Appl. Spectros. (USSR)*, vol. 40, pp. 657–660, 1984.
- [181] S.-C. Sheng, "Optical-axis perturbation singularity in an out-of-plane ring resonator," *Opt. Lett.*, vol. 19, pp. 683–685, May 15, 1994.
- [182] M. McGeoch, "Prism reflectors for high-power lasers," *Opto-Electron.*, vol. 2, pp. 85–89, 1970.
- [183] I. D. Carr and D. C. Hanna, "A prism reflector of anti-resonant ring configuration," *Opt. Quantum Electron.*, vol. 15, pp. 447–450, 1983.
- [184] A. E. Siegman, "An antiresonant ring interferometer for coupled laser cavities, laser output coupling, mode locking, and cavity dumping," *IEEE J. Quantum Electron.*, vol. QE-9, pp. 247–250, Feb. 1973.
- [185] R. Trutna and A. E. Siegman, "Laser cavity dumping using an antiresonant ring," *IEEE J. Quantum Electron.*, vol. QE-13, pp. 955–962, Dec. 1977.
- [186] G. B. Michelangeli, E. Penco, G. Guilianni, and E. Palange, "Q-switching and cavity dumping of a high-power cw Nd-YAG laser by means of a novel electro-optic configuration," *Opt. Lett.*, vol. 11, pp. 360–362, June 1986.
- [187] O. G. Okhotnikov and F. M. Araujo, "Cavity dumping of fiber lasers by phase-modulated optical loop mirrors," *Opt. Lett.*, vol. 21, pp. 57–58, Jan. 1, 1996.
- [188] A. E. Siegman, "Passive mode locking using an antiresonant-ring laser cavity," *Opt. Lett.*, vol. 6, pp. 334–335, July 1981.
- [189] H. Vanherzeele, J. L. Van Eck, and A. E. Siegman, "Colliding pulse mode locking of a Nd:YAG laser with an antiresonant ring structure," *Appl. Opt.*, vol. 20, pp. 3484–3486, Oct. 15, 1981.
- [190] T. Norris, T. Sizer, and G. Mourou, "Generation of 90-fs pulses with a CPM laser pumped by a frequency-doubled CW mode-locked Nd:YAG," *IQEC*, 1984.
- [191] H. Vanherzeele, J.-C. Diels, and R. Torti, "Tunable passive colliding-pulse mode locking in a linear dye laser," *Opt. Lett.*, vol. 9, pp. 549–551, Dec. 1984.
- [192] T. F. Carruthers and I. N. Duling III, "Passive laser mode locking with an antiresonant nonlinear mirror," *Opt. Lett.*, vol. 15, pp. 804–806, July 15, 1990.
- [193] K. Naganuma and I. K. Mogi, "50-fs pulse generation directly from a colliding-pulse mode-locked Ti:sapphire laser using an antiresonant ring mirror," *Opt. Lett.*, vol. 16, pp. 738–740, May 15, 1991.
- [194] A. Agnesi, G. C. Reali, and G. Gabetta, "The effect of transverse multimode oscillation in passively mode-locked solid-state lasers," *Opt. Commun.*, vol. 88, pp. 54–58, Mar. 1, 1992.
- [195] C. Borde, G. Camy, B. Decomps, and L. Pottier, "Saturated dispersion experiment in iodine with an argon laser," *C. R. Acad. Sci.*, vol. 277, p. 381, 1973.
- [196] K. Otsuka, "Nonlinear antiresonant ring interferometer," *Opt. Lett.*, vol. 8, pp. 471–473, Sept. 1983.
- [197] Y. Li, G. Eichmann, and R. R. Alfano, "Multistable Fabry–Perot resonator with an active Sagnac interferometer as its retro-reflector," *Opt. Commun.*, vol. 61, pp. 75–79, Jan. 1, 1987.
- [198] R. Trebino and C. C. Hayden, "Antiresonant-ring transient spectroscopy," *Opt. Lett.*, vol. 16, pp. 493–495, Apr. 1, 1991.
- [199] J. Yu and P. Jeppesen, "Wavelength conversion by use of four-wave mixing in a novel optical loop configuration," *Opt. Lett.*, vol. 25, pp. 395–397, Mar. 15, 2000.
- [200] M. C. Farries and D. N. Payne, "Optical fiber switch employing a Sagnac interferometer," *Appl. Phys. Lett.*, vol. 55, pp. 25–26, July 3, 1989.
- [201] W. S. Wong *et al.*, "Self-switching of optical pulses in dispersion-imbalance nonlinear loop mirrors," *Opt. Lett.*, vol. 22, pp. 1150–1152, Aug. 1, 1997.
- [202] K.-X. Sun, M. M. Fejer, E. Gustafson, and R. L. Byer, "Sagnac interferometer for gravitational-wave detection," *Phys. Rev. Lett.*, vol. 76, pp. 3053–3056, Apr. 22, 1996.
- [203] K.-X. Sun, E. K. Gustafson, M. J. Fejer, and R. L. Byer, "Polarization-based balanced heterodyne detection method in a Sagnac interferometer for precision phase measurement," *Opt. Lett.*, vol. 22, pp. 1359–1361, Sept. 1, 1997.
- [204] K.-X. Sun, M. J. Fejer, E. K. Gustafson, and R. L. Byer, "Balanced heterodyne signal extraction in a postmodulated Sagnac interferometer at low frequency," *Opt. Lett.*, vol. 22, pp. 1485–1487, Oct. 1, 1997.
- [205] E. J. Heller, "Bound-state eigenfunctions of classically chaotic Hamiltonian systems: Scars of periodic orbits," *Phys. Rev. Lett.*, vol. 53, pp. 1515–1518, Oct. 15, 1984.
- [206] S. Sridhar, "Experimental observation of scarred eigenfunctions of chaotic microwave cavities," *Phys. Rev. Lett.*, vol. 67, pp. 785–787, Aug. 12, 1991.
- [207] S. Sridhar and E. J. Heller, "Physical and numerical experiments on the wave mechanics of classically chaotic system," *Phys. Rev. A*, vol. 46, pp. R1728–R1731, Aug. 15, 1992.
- [208] E. J. Heller and S. Tomsovic, "Postmodern quantum mechanics," *Phys. Today*, vol. 46, pp. 38–46, July 1993.
- [209] S. Biellak, "Reactive ion etched semiconductor diode laser resonators," Ph. dissertation, Center for Integrated Systems, Stanford Univ., Stanford, CA, Aug. 1995.
- [210] T. Fukushima, S. A. Biellak, Y. Sun, and A. E. Siegman, "Beam propagation behavior in a quasistadium laser diode," *Opt. Expr.*, vol. 2, pp. 21–28, Jan. 19, 1998.
- [211] C. Gmachl *et al.*, "High-power directional emission from microlasers with chaotic resonators," *Science*, vol. 280, pp. 1556–1564, June 5, 1998.
- [212] K. Patorski, "The self imaging phenomenon and its applications," in *Progress in Optics*, E. Wolf, Ed. Amsterdam, The Netherlands: North-Holland, vol. 27, pp. 1–108.
- [213] P. P. Pashinin, *Formation and Control Wavefronts*. Commack, NY: Nova, 1989.
- [214] P. A. Belanger, R. L. Lachance, and C. Pare, "Super-Gaussian output from a CO₂ laser by using a graded-phase mirror resonator," *Opt. Lett.*, vol. 17, pp. 739–741, May 15, 1992.
- [215] C. Pare and P. Belanger, "Custom laser resonators using graded-phase mirrors," *IEEE J. Quantum Electron.*, vol. QE-28, pp. 355–362, Jan. 1992.
- [216] —, "Custom laser resonators using graded-phase mirrors: Circular geometry," *IEEE J. Quantum Electron.*, vol. QE-30, pp. 1141–1148, Apr. 1994.
- [217] R. Van Neste, C. Pare, R. L. Lachance, and P. Belanger, "Graded-phase mirror resonator with a super-Gaussian output in a CW CO₂ laser," *IEEE J. Quantum Electron.*, vol. QE-30, pp. 2663–2669, Nov. 1994.

- [218] J. R. Leger, D. Chen, and Z. Wang, "Diffractive optical element for mode shaping of a Nd:YAG laser," *Opt. Lett.*, vol. 19, pp. 108–110, 1994.
- [219] J. R. Leger, D. Chen, and K. Dai, "High modal discrimination in a Nd:YAG laser resonator with internal phase gratings," *Opt. Lett.*, vol. 19, pp. 1976–1978, Dec. 1, 1994.
- [220] D. Chen, Z. Wang, and J. R. Leger, "Measurements of the modal properties of a diffractive optic graded phase resonator," *Opt. Lett.*, vol. 20, pp. 663–665, 1995.
- [221] J. R. Leger, D. Chen, and G. Mowry, "Design and performance of diffractive optics for custom laser resonators," *Appl. Opt.*, vol. 34, pp. 2498–2509, 1995.
- [222] G. Mowry and J. R. Leger, "Large-area, single transverse mode semiconductor laser with diffraction-limited super-Gaussian output," *Appl. Phys. Lett.*, vol. 66, pp. 1614–1616, 1995.
- [223] S. Makki and J. R. Leger, "Solid-state laser resonators with diffractive optic thermal aberration correction," *IEEE J. Quantum Electron.*, vol. QE-35, pp. 1075–1085, June 1999.
- [224] A. E. Siegman, "How to (maybe) measure laser beam quality," in *DPSS Lasers: Applications and Issues*, M. W. Dowley, Ed. Washington, D.C.: Optical Society of America, 1998, pp. 184–199.
- [225] Y. K. Danileiko and V. A. Lobochev, "New rotating-field resonator for lasers," *Kvantovaya Elektron., Mosk.*, vol. 1, pp. 688–690, Mar. 1974.

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