International Journal of Modern Physics B, Vol. 11, No. 16 (1997) 1913–1924
© World Scientific Publishing Company

HOMODYNE PHOTOCURRENT, SYMMETRIES IN PHOTON MIXING AND NUMBER STATE SYNTHESIS

MATTEO G. A. PARIS
Arbeitsgruppe "Nichtklassiche Strahlung"
Max-Planck-Gesellchaft - Humboldt Universität zu Berlin
Rudower Chaussee5 - 12484 Berlin - Germany

Received 4 September 1996

Lossless beam splitter has been studied by the exact diagonalization of the homodyne photocurrent. The probability of totally symmetric and totally antisymmetric output from a balanced beam splitter fed by arbitrary number states has been evaluated. Efficiency of photon mixing to synthesize highly excited number states has been also discussed.

1. Introduction

The beam splitter is a simple device, however providing a deep insight into the quantum nature of the electromagnetic field. Many experiments on the wave-particle duality has been indeed performed by mixing weak light beams in such a device.[1] In a beam splitter two different spatial modes of the radiation field are linearly coupled through the interaction with a medium showing first order susceptibility. Such an optical coupler can be realized by a dielectric interface, a passive interferometer and also by *all-fiber* devices.[2–4]

Balanced beam splitter is defined as one showing equal values for the transmission and reflection coefficients. It has been received much theoretical attention.[5,6] Apart from the appeal of symmetry this is mainly due to the fact that a balanced beam splitter is the basic ingredient of the homodyne detection scheme.[7–11] A balanced beam splitter with input arms fed by one photon each shows totally antisymmetric output, namely both photons exit from the same arm with unit probability. This is useful in synthesizing number states with two photons starting from number states with one photon. The same process with highly excited number states has not unit probability. Thus the efficiency of photon mixing as a process to synthesize number states decreases. The evaluation of synthesis efficiency requires the computation of higher symmetries in photon mixing.

In this paper we approach the problem of beam splitter output starting from homodyne detection. First we derive an exact diagonalization for the homodyne

E-mail:paris@photon.fta-berlin.de

photocurrent and then, using this result, the probability of totally symmetric and totally antisymmetric output from a beam splitter fed by arbitrary number states has been evaluated. Finally, a conditioned measurement scheme has been suggested for producing highly excited number states by photon mixing. Its efficiency when implemented with currently available detectors has been also discussed.

We report a brief description of beam splitter and homodyne detection in Sec. 2 whereas Sec. 3 is devoted to the diagonalization of the homodyne photocurrent. The occurrence of totally symmetric and totally antisymmetric output and the number states synthesis efficiency are discussed in Secs. 4 and 5. Some concluding remarks are contained in Sec. 6.

2. Beam Splitter and Homodyne Detection

A schematic diagram of a beam splitter is reported in Fig. 1. Such an optical device can be easily realized by means of a linear medium where the polarization vector

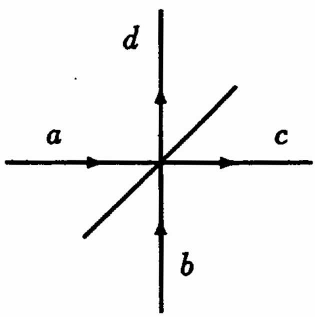

Fig. 1. Schematic diagram of a beam splitter.

is simply proportional to incoming field $\hat{\mathbf{P}} = \chi\hat{\mathbf{E}}$, $\chi \equiv \chi^{(1)}$ being the first order (linear) susceptibility. We consider the incoming field excited only in the relevant spatial modes a and b (at the same frequency ω),

$$\hat{\mathbf{E}}(\mathbf{r},t) = i\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\left[(a+b)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega\, t)} + \text{h.c.}\right] . \tag{2.1}$$

The interaction Hamiltonian contain only the resonant terms

$$\hat{H}_I = -\hat{\mathbf{P}}\cdot\hat{\mathbf{E}} = -\chi\widehat{\mathbf{E}^2} = \frac{\chi\hbar\omega}{2\epsilon_0 V}\left(a^\dagger b + ab^\dagger\right) , \tag{2.2}$$

whereas the evolution operator (in the interaction picture) of the whole device is expressed as

$$\hat{U} = \exp\left[i \arctan\sqrt{\frac{1-\tau}{\tau}}\left(a^\dagger b + ab^\dagger\right)\right] \tag{2.3}$$

where τ given by

$$\tau = \left[1 + \tan^2\left(\frac{\chi\hbar\omega}{2\epsilon_0 V}\right)\right]^{-1}, \tag{2.4}$$

represents the transmissivity of the beam splitter. The Heisenberg evolution equations for field modes

$$\begin{pmatrix} c \\ d \end{pmatrix} = \hat{U}^\dagger \begin{pmatrix} a \\ b \end{pmatrix} \hat{U}, \tag{2.5}$$

can be exactly solved leading to

$$\begin{cases} c = -i\tau^{1/2}a + (1-\tau)^{1/2}b \\ d = i(1-\tau)^{1/2}a + \tau^{1/2}b \end{cases}. \tag{2.6}$$

In Eqs. (2.6) the phase of the mode a (or equivalently that one of b) is arbitrary. A rotation of the phase frame by $\frac{3}{2}\pi$ leads to

$$\begin{cases} c = \tau^{1/2}a + (1-\tau)^{1/2}b \\ d = -(1-\tau)^{1/2}a + \tau^{1/2}b \end{cases}, \tag{2.7}$$

which can be obtained from Eqs. (2.6) by the substitution $a \to -ia$. In the following we prefer the form (2.7) as they look closer to their classical counterpart.

A schematic diagram of a homodyne detector is depicted in Fig. 2.

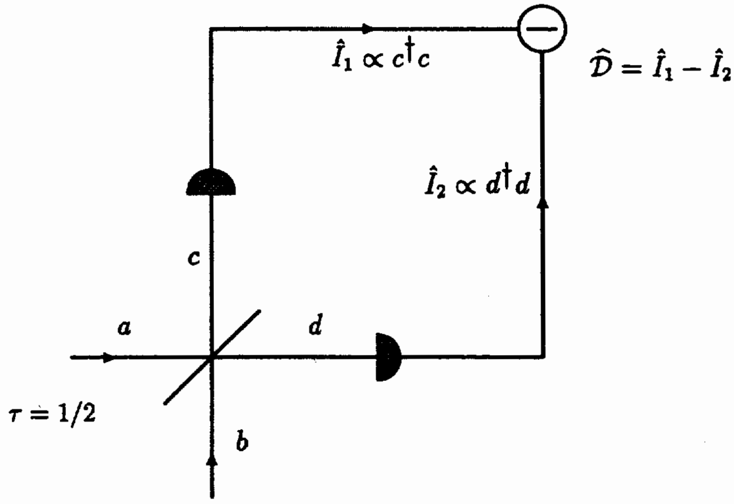

Fig. 2. Schematic diagram of a homodyne detector.

Using Eqs. (2.7) we can express the homodyne photocurrent in terms of the input modes

$$\hat{\mathcal{D}} = (2\tau - 1)(a^\dagger a - b^\dagger b) + 2\sqrt{\tau(1-\tau)}(a^\dagger b + ab^\dagger), \tag{2.8}$$

which reduces to

$$\widehat{\mathcal{D}} = a^\dagger b + ab^\dagger \,. \tag{2.9}$$

for balanced ($\tau = 1/2$) beam splitter. Balanced homodyne detector is often used to determine the statistics of one-mode field quadrature. In brief, if the mode b is placed in a highly excited coherent state (a quasiclassical state) we can substitute in Eq. (2.9) the field mode b by the complex amplitude $z = |z| \exp\{i\varphi\}$, so that the reduced photocurrent

$$\mathcal{I} = \frac{\widehat{\mathcal{D}}}{2|z|} = \frac{1}{2}(a^\dagger e^{i\varphi} + ae^{-i\varphi}) \tag{2.10}$$

traces exactly the field quadrature $\hat{a_\varphi}$ of the mode a. Here we shall consider a more general situation, in which also the mode b can be placed in quantum state. Thus, the homodyne photocurrent $\widehat{\mathcal{D}}$ is a two-mode operator acting on the whole Hilbert space $\mathcal{H}_a \otimes \mathcal{H}_b$, the direct product of the Hilbert spaces of the two modes. In the next section we will report the exact diagonalization for the operator $\widehat{\mathcal{D}}$ in terms of two-modes states.

3. Diagonalizing Homodyne Photocurrent

The basic tool of our calculations is represented by the unitary transformation described by the operator

$$\widehat{\mathcal{U}}_\lambda = e^{\lambda(a^\dagger b - ab^\dagger)} \,. \tag{3.1}$$

The transformation in Eq. (3.1) acts on the total Hilbert space $\mathcal{H} = \mathcal{H}_\dashv \otimes \mathcal{H}_\lfloor$ which is the direct product of the two Hilbert spaces describing the two modes respectively. Using the operatorial relations

$$e^{\hat{A}}\, \hat{B}\, e^{-\hat{A}} = \hat{B} + [\hat{A}, \hat{B}] + \frac{1}{2}[\hat{A}, [\hat{A}, \hat{B}]] + \cdots + \\ + \frac{1}{n!} \underbrace{[\hat{A}, [\hat{A}, \ldots, [\hat{A}, \hat{B}]]]}_{n \text{ times}} + \cdots \,, \tag{3.2}$$

one obtains the action of $\widehat{\mathcal{U}}_\lambda$ on the single-mode Bose operators which results in the following linear combinations[a]

$$\begin{aligned} \widehat{\mathcal{U}}_\lambda^\dagger \; a \; \widehat{\mathcal{U}}_\lambda &= a\cos\lambda + b\sin\lambda \\ \widehat{\mathcal{U}}_\lambda^\dagger \; b \; \widehat{\mathcal{U}}_\lambda &= -a\sin\lambda + b\cos\lambda \,. \end{aligned} \tag{3.3}$$

Let us now consider the particular value $\lambda = \pi/4$ which leads to the symmetric combinations

[a]We use the notations a and b for the single-mode Bose operators. This is for symplicity as no ambiguities can occur. A more formal notation would be $a \otimes \hat{1}_b$ and $\hat{1}_a \otimes b$ with $\hat{1}_i$, $i = a, b$ denoting identity operators in the single-mode Hilbert space.

$$\lambda = \frac{\pi}{4} \Longrightarrow \begin{cases} \widehat{\mathcal{U}}^{\dagger}_{\pi/4}\, a\, \widehat{\mathcal{U}}_{\pi/4} = \frac{1}{\sqrt{2}}(b+a) \\ \widehat{\mathcal{U}}^{\dagger}_{\pi/4}\, b\, \widehat{\mathcal{U}}_{\pi/4} = \frac{1}{\sqrt{2}}(b-a) \end{cases} . \tag{3.4}$$

In this way it is possible to reexpress the operator $\widehat{\mathcal{D}}$ as

$$\widehat{\mathcal{D}} = \widehat{\mathcal{U}}^{\dagger}_{\pi/4}(a^{\dagger}a - b^{\dagger}b)\colon \widehat{\mathcal{U}}_{\pi/4}\,, \tag{3.5}$$

which will become very useful in its diagonalization. Indeed, the eigenvalues problem for the operator $\widehat{\mathcal{D}}$

$$\widehat{\mathcal{D}}\,|\delta\rangle\rangle = d\,|\delta\rangle\rangle\,, \tag{3.6}$$

can be rewritten in terms of the eigenvalues problem of the operator $a^{\dagger}a - b^{\dagger}b$

$$(a^{\dagger}a - b^{\dagger}b)\,|\nu\rangle\rangle = d\,|\nu\rangle\rangle\,, \tag{3.7}$$

where

$$|\nu\rangle\rangle = \widehat{\mathcal{U}}_{\pi/4}\,|\delta\rangle\rangle\,. \tag{3.8}$$

In Eqs. (3.6), (3.7) and (3.8) the symbol $|\cdot\rangle\rangle$ denotes a state in the total Hilbert space $\mathcal{H}$.

The eigenvalues problem in Eq. (3.7) can be easily solved by the Fock description of the two-mode Hilbert space. We easily find

$$|\nu^{(n)}\rangle\rangle = \begin{cases} |n+d\rangle_a \otimes |n\rangle_b\,, & d \in \mathbf{Z}^+\,, \\ |n\rangle_a \otimes |n\rangle_b\,, & d = 0\,, \\ |n\rangle_a \otimes |n-d\rangle_b\,, & d \in \mathbf{Z}^-\,, \end{cases} \tag{3.9}$$

where $|q\rangle_{a,b}$, $q \in \mathbf{N}$ denote the eigenstates of the number operators $\hat{n}_a = a^{\dagger}a$ in $\mathcal{H}_a$ and $\hat{n}_b = b^{\dagger}b$ in $\mathcal{H}_b$ respectively. In Eq. (3.9) $\mathbf{Z}^{\pm} = \pm\mathbf{N}$ denotes the positive and negative sets of integers respectively.

The eigenvalue d has countable degeneracy, corresponding to one-integer-parameter set $|\nu^{(n)}\rangle\rangle$ of eigenstates. In order to solve the original eigenvalues problem we have to compute their transformation under the action of the operator $\widehat{\mathcal{U}}_{\pi/4}$, that is from Eq. (3.8)

$$|\delta^{(n)}\rangle\rangle = \widehat{\mathcal{U}}^{\dagger}_{\pi/4}\,|\nu^{(n)}\rangle\rangle\,. \tag{3.10}$$

With the aim of evaluating Eq. (3.10) we consider the Schwinger two bosons realization of SU(2) Lie algebra[13]

$$\begin{gathered}[\hat{J}_+, \hat{J}_-] = 2\hat{J}_3\,, \qquad [\hat{J}_3, \hat{J}_\pm] = \pm\hat{J}_\pm \\ \hat{J}_+ = ab^{\dagger}\,, \qquad \hat{J}_- = a^{\dagger}b\,, \qquad J_3 = \frac{1}{2}(b^{\dagger}b - a^{\dagger}a)\,. \end{gathered} \tag{3.11}$$

As can easily recognized from Eq. (3.1) the operator $\widehat{\mathcal{U}}$ has the form of a SU(2) displacement

$$\widehat{\mathcal{U}}_{\pi/4} = \exp\left\{\frac{\pi}{4}(\hat{J}_+ - \hat{J}_-)\right\}\,. \tag{3.12}$$

This means that it can be disentangled using SU(2) Baker–Haussdorff formula

$$\exp\{\xi\hat{J}^{+}-\bar{\xi}\hat{J}^{-}\}=\exp\{\zeta\hat{J}^{+}\}\exp\{\beta\hat{J}^{3}\}\exp\{-\bar{\zeta}\hat{J}^{-}\}$$
$$\zeta=\frac{\xi}{|\xi|}\tan|\xi| \tag{3.13}$$
$$\beta=\ln(1+|\zeta|^{2}),$$

leading to the expression

$$\widehat{\mathcal{U}}^{\dagger}_{\pi/4}=\exp\{ab^{\dagger}\}\;\exp\{\log\sqrt{2}(b^{\dagger}b-a^{\dagger}a)\}\;\exp\{-a^{\dagger}b\}\,. \tag{3.14}$$

We now substituting Eq. (3.14) in Eq. (3.10). Upon expanding exponential factors, and using the action of the powers of Bose operators on number eigenstates,

$$a^{p}|q\rangle=\Theta(q-p)\sqrt{\frac{q!}{(q-p)!}}\;|q-p\rangle$$
$$a^{\dagger p}|q\rangle=\sqrt{\frac{(q+p)!}{q!}}\;|q+p\rangle \tag{3.15}$$
$$\exp\{\hat{N}p\}\;|q\rangle=\exp\{qp\}\;|q\rangle\,,$$

we obtain an explicit formula for the eigenstates

$$|\delta^{(n)}\rangle=\sum_{k=0}^{n}\sum_{l=0}^{n+d+k}(-)^{k}\sqrt{\binom{n}{k}\binom{n+d+k}{k}}\;\frac{1}{2^{k+d/2}}$$
$$\times\sqrt{\binom{n+d+k}{l}\binom{n-k-l}{l}}$$
$$\times\;|n+d+k-l\rangle_{a}\otimes|n-k+l\rangle_{b}\,. \tag{3.16}$$

In Eq. (3.15) $\Theta(x)$ denotes the step function which is unit for nonnegative argument and zero otherwise.

We now consider the following transformation on the discrete set of indices entering in Eq. (3.16):

$$\begin{cases} s=l+k \\ t=l-k \end{cases}\Longleftrightarrow\begin{cases} l=\frac{1}{2}(s+t) \\ k=\frac{1}{2}(s-t) \end{cases}. \tag{3.17}$$

It induces a "double spacing"on $s-t$ grid, so that the rule for sum transformation reads as follows

$$\sum_{k}\sum_{l}\longrightarrow\sum_{t}\sum_{s}{}^{(*t)}\,. \tag{3.18}$$

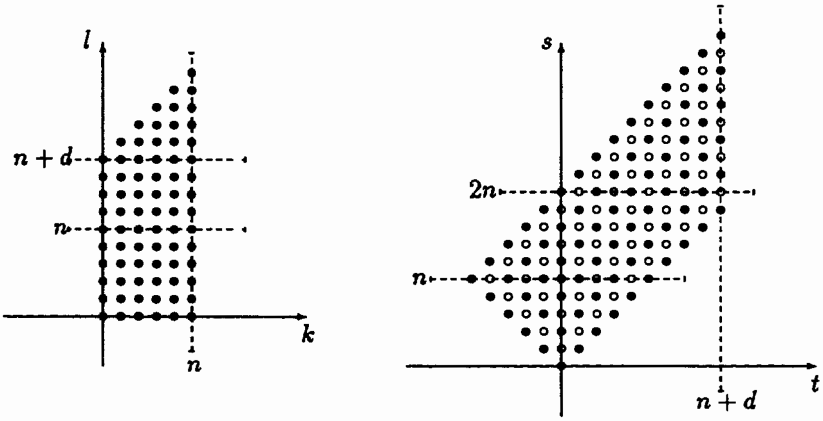

Fig. 3. The transformation from the k–l grid to the s–t grid is not isometric. A doubling is, in fact, induced on the latter. In the s–t plane the filled circles indicate the points which belong to the original grid, whereas empty circles denote spurius pairs. The introduction of alternate summation is thus needed, as a discrete counterpart of Jacobian determinant.

In Eq. (3.18) the symbol $\sum_s{}^{(*t)}$ denotes "alternate" summmation, that is, summation over even s when t is even and vice versa. We illustrate this rule in Fig. 3. The summations in Eq. (3.16) become

$$\sum_{k=0}^{n}\sum_{l=0}^{n+d+k} = \sum_{t=-n}^{n}\sum_{s=|t|}^{n}{}^{(*t)} + \sum_{t=-n+1}^{n+d}\sum_{s=\mathcal{M}_{nt}}^{t+2n}{}^{(*t)}. \tag{3.19}$$

where

$$\mathcal{M}_{nt} = \max\{1+n, t\}. \tag{3.20}$$

From Eqs. (3.16), (3.17) and (3.19) we are now able to write a closed formula for the eigenstates of the operator $\widehat{\mathcal{D}}$.
For positive values of d we have

$$|\delta^{(n)}\rangle\rangle = \sum_{t=-n}^{n+d} C_t^{n,d}\,|n+d-t\rangle_a \otimes |n+t\rangle_b \tag{3.21}$$

whereas for negative values of d the same formula holds, however we have to perform the shift $n \to n-d$.
The coefficients in the expansion (3.21) are given by (positive d)

$$\begin{aligned} C_t^{n,d} &= \sum_{s=|t|}^{n}{}^{(*t)}\ (-)^{\frac{s-t}{2}}\,K_s^{n,d,t}\,2^{\frac{t-d-s}{2}} \qquad t\in[-n,n] \\ C_t^{n,d} &= \sum_{s=\mathcal{M}_{nt}}^{t+2n}{}^{(*t)}\,(-)^{\frac{s-t}{2}}\,K_s^{n,d,t}\,2^{\frac{t-d-s}{2}} \qquad t\in[-n+1,n+d] \end{aligned} \tag{3.22}$$

where

$$K_s^{n,d,t} = \sqrt{\binom{n}{(s-t)/2}\binom{n+d+(s-t)/2}{(s-t)/2}}$$
$$\sqrt{\binom{n+d+(s-t)/2}{(s+t)/2}}n+t(s+t)/2\,. \tag{3.23}$$

Again, the above formulas are valid also for negative d after the substitution $n \to n-d$. The operator $\widehat{\mathcal{D}}$ can be written as

$$\widehat{\mathcal{D}} = \sum_{d\in\mathbf{Z}} d\,\widehat{\mathbf{M}}_d\,. \tag{3.24}$$

$\widehat{\mathbf{M}}_d$ being its spectral (operator valued) measure[12]

$$\widehat{\mathbf{M}}_d = \sum_{n\in\mathbf{N}} |\delta^{(n)}\rangle\rangle\langle\langle\delta^{(n)}|\,. \tag{3.25}$$

Probability distribution for the photon difference is obtained by trace operation

$$P(d) = \mathrm{Tr}\left\{\hat\rho\,\widehat{\mathbf{M}}_d\right\} = \sum_{n\in\mathbf{N}} \langle\langle\delta^{(n)}|\hat\rho|\delta^{(n)}\rangle\rangle\,, \tag{3.26}$$

$\hat\rho \in \mathcal{H}$ being the total density matrix (not necessarily factorized). For a pure factorized state at the input, namely

$$|\psi\rangle_{in} = \sum_{mn} k_n^1\, k_m^2\, |n\rangle_a \otimes |m\rangle_b\,, \tag{3.27}$$

Eq. (3.26) reduces to

$$P(d) = \sum_{n\in\mathbf{N}} \left|\sum_{t=-n}^{n+d} C_t^{n,d}\, k^1_{n+d-t} k^2_{n+t}\right|^2 . \tag{3.28}$$

Still Eq. (3.28) is valid for positive d, whereas for negative d the shift $n \to n-d$ has to be performed on the index n.

4. Higher Symmetries in Photon Mixing

In this section we use the diagonalization of the homodyne photocurrent to extract information on the statistics of photon mixing. First, we present the result for the case of photon splitting. We consider a mode excited with N photons mixed with the vacuum by a balanced beam splitter, that is we have at the input

$$|\psi\rangle_{\rm in} = |N\rangle_a \otimes |0\rangle_b\,.$$

The whole set of the possible output states can be written as

$$|\psi\rangle_{\rm out} = |N\rangle_a \otimes |N-d\rangle_b\,, \tag{4.1}$$

where

$$d \in \mathbf{Z} \text{ and } |d| < N\,.$$

Remarkably their probabilities (the probabilities of their occurences) are governed by only one coefficient $C_t^{n,d}$. As it can be easily checked from Eq. (3.28) only the value of d leading to even $(N \pm d)$ has nonzero probability. For these values the probability distribution reads as

$$P(d) = \left|C_{(d-N)/2}^{(N-d)/2,d}\right|^2 = \frac{1}{2^N}\frac{N!}{[(N-d)/2]![(N+d)/2]!}\,\Theta(N-|d|)\,, \qquad (4.2)$$

$\Theta(x)$ being the step function.

Let us now consider the more general case, where both the input modes are excited in arbitrary number states

$$|\psi\rangle_{\text{in}}+ = |N_1\rangle_a \otimes |N_2\rangle_b\,.$$

We have totally asymmetric output when, after interaction, one of the two states

$$|\psi\rangle_{\text{out}} = |0\rangle_a \otimes |N_1 + N_2\rangle_b$$

or

$$|\psi\rangle_{\text{out}} = |N_1 + N_2\rangle_a \otimes |0\rangle_b$$

is emerging from the beam splitter.

The probability $P^A(N_1, N_2)$ of such an event is governed by two coefficients $C_t^{n,d}$

$$P^A(N_1, N_2) = \left|C_{N_2}^{0,N_1+N_2}\right|^2 + \left|C_{-N_1}^{0,-N_1-N_2}\right|^2\,. \qquad (4.3)$$

corresponding to the two possible values $d = \pm(N_1 + N_2)$. A direct inspection of Eqs. (3.22) and (3.23) shows that

$$P^A(N_1, N_2) = \frac{(N_1+N_2)!}{N_1!N_2!}\frac{1}{2^{N_1+N_2-1}}\,. \qquad (4.4)$$

In Fig. 4b we show the probability of totally antisymmetric output as a function of the photons number in the two input modes.

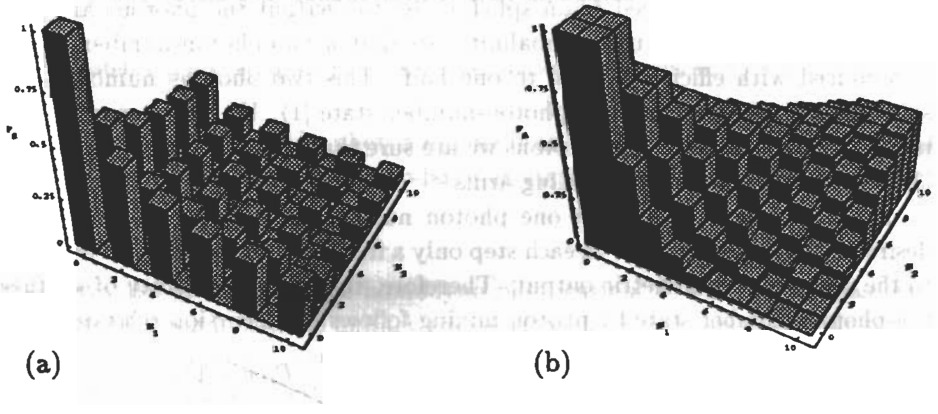

Fig. 4. Probabilities of totally antisymmetric (a), and totally symmetric (b) output from a balanced beam splitter as a function of the photons number in the two input modes.

The opposite case of totally symmetric output is realized when the output state emerging from the beam splitter is given by

$$|\psi\rangle_{out} = \left|\frac{1}{2}(N_1+N_2)\right\rangle_a \otimes \left|\frac{1}{2}(N_1+N_2)\right\rangle_b \,.$$

This corresponds to a zero difference photocurrent and thus to a probability given by

$$P^S(N_1,N_2) = \left|C^{1/2(N_1+N_2),0}_{1/2(N_2-N_1)}\right|^2 , \tag{4.5}$$

which can be easily computed numerically from Eqs. (3.22) and (3.23). Obviously, probability in Eq. (4.5) is non zero only for even (N_1+N_2). In Fig.4a we show the probability of totally symmetric output as a function of the photons number in the two input modes.

5. Number State Synthesis by Photon Mixing

As can be easily pointed out from Eq. (4.1), is the following. Once the total number of photons (N_1+N_2) impinged into the beam splitter is known, a measurement of the photon number in one arm gives unambiguos information about the number of photons in the second arm. In other words, the quantum correlations between the two output modes can be used to uniquely characterize one of them without perturbing it. This is a kind of a conditioned state preparation and we call it a *number states synthesis* by photon mixing. Unfortunately, customary photocounters can effectively distinguish only whether or not photons are present,[14] without any information on the precise number of recorded photons. Thus, only when the output of the beam splitter is totally antisymmetric we can, after a photocounting process in one arm of the beam splitter, predict with certainty which kind of number states is exiting from the other arm. Moreover, only one-photon number state $|1\rangle$ can be produced with high reliability from a parametric down conversion process.[18] The synthesis process is thus designed as follows (see also Fig. 5). Two single photons are firstly mixed by a balanced beam splitter. At the output the photons are packed in the same mode with unit probability, so that a two-photon-number state $|2\rangle$ is produced with efficiency equal to one half. This two photons number state is subsequently mixed with a one-photon-number state $|1\rangle$. If a photocounter placed in the probing arm counts no photons we are sure that a three-photon-number state $|3\rangle$ is travelling down the remaining arm.

Further mixings with other one photon number states eventually lead to the desired excitation number.[15] In each step only a fraction $\frac{1}{2}P^A(n,1)$ of the events lead to the desired antisymmetric output. Therefore, the total probability of synthesize a n-photons number state by photon mixing follows the recursion relation

$$P_s(n) = P^A(n-1,1)P_s(n-1) = \frac{n}{2^{n-1}}P_s(n-1)\,, \tag{5.1}$$

where $P^A(n,1)$ is given in Eq. (3.28) and $P_s(1)=1$. In Fig. 6 we report $P_s(n)$ as a function of n. This probability decreases rapidly as a function of n, thus indicating

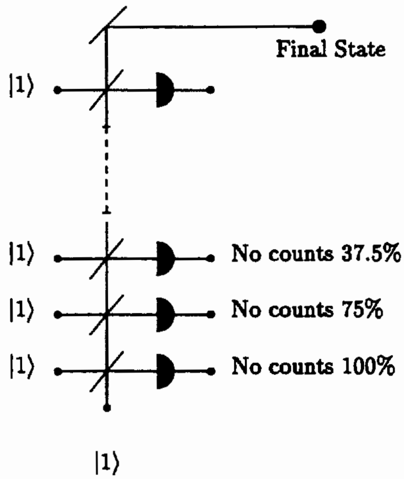

Fig. 5. Schematic diagram of photon mixing device to synthesize higher excited number states starting from one photon state. All beam splitters are balanced and efficiencies at each step are from Eq. (5.1).

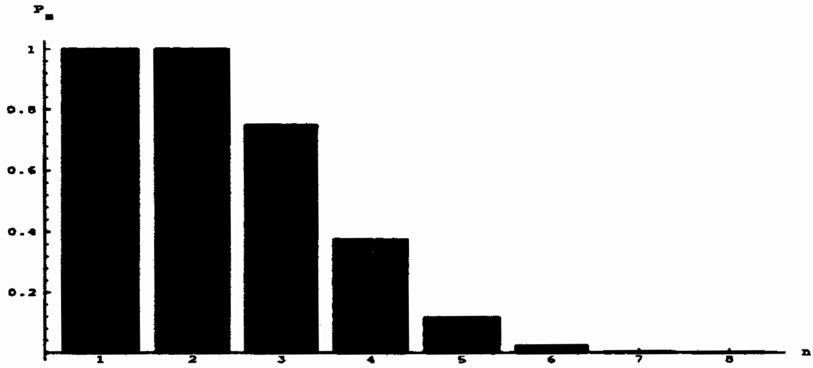

Fig. 6. Probability of producing a n-photons number state (synthesis efficiency) by photon mixing.

that number state synthesis by photon mixing is an effective method for preparation of number states only when a not-too-large excitation number is needed.

6. Conclusion

The eigenvalues problem for the homodyne photocurrent $\widehat{D} = a^\dagger b + ab^\dagger$ has been exactly solved upon a suitable unitary transformation. Using this result the occurrence of totally symmetric and totally antisymmetric output from a beam splitter fed by arbitrary number states has been evaluated. The efficiency of photon mixing in synthesizing highly excited number states starting from single photons has also been discussed. This quantity rapidly decreases with increasing excitations of the required number state, becoming vanishingly small for $n \geq 10$. However, this negative result should be compared with that one coming from number synthesis by nonlinear interaction.[19] In that case the bare efficiency of the process is higher than in the present linear scheme. However, there is an intrinsic inefficiency, related to high order susceptibility process, which again makes this synthesis scheme a very low rate one.

The homodyne photocurrent diagonalization can be also be useful in recovering information about the mode a viewed as an unknown signal mode probed by the mode b. In such a quantum state measurement[16,17] scheme Eq. (3.28) is suitable to evaluate the different statistic coming from different probe mode. Work along this line is in progress and results will be reported elsewhere.

Acknowledgments

I would thank Prof. Harri Paul for his kind hospitality in the group 'Nichtklassiche Strahlung' of Max-Planck society. I would also thank Valentina De Renzi, Ole Steuernagel, Gabriel Drobny, Alexei Chizhov and Alfred Wünsche for profitable discussions. This work has been partially supported by the University of Milan by a scholarship for postgraduate studies in foreign countries.

References

1. C. K. Hong, Z. Y. Ou and L. Mandel, *Phys. Rev Lett.* **59**, 2044 (1987); J. G. Rarity and P. R. Tapster, *J. Opt. Soc. Am.* **B6**, 1221 (1989).
2. B. Yurke, S. L. McCall and J. R. Klauder, *Phys. Rev.* **A33**, 4033 (1986).
3. J. Janszky, C. Sibilia, M. Bertolotti and Y. Yushin, *J. Mod. Opt.* **35**, 1757 (1988); J. Janszky, P. Adam, M. Bertolotti and C. Sibilia, *Quantum Opt.* **4**, 163 (1992).
4. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, (Cambridge University Press, 1995).
5. H. Fearn, *Quantum Opt.* **2**, 103 (1990); H. Fearn and R. Loudon, *Opt. Comm.* **64**, 485 (1987).
6. R. A. Campos, B. E. A. Saleh and M. C. Teich, *Phys. Rev.* **A40**, 1371 (1989).
7. H. P. Yuen and V. W. S. Chan, *Opt. Lett.* **8**, 177 (1983).
8. M. G. A. Paris, *Mod. Phys. Lett.* **B9**, 1141 (1995).
9. M. G. A. Paris, *Phys. Lett.* **A217**, 78 (1996).
10. G. M. D'Ariano, *Int. J. Mod. Phys.* **B6**, 1291 (1992).
11. U. Leonhardt, *Phys. Rev.* **A48**, 3265 (1993).
12. P. Busch and P. J. Lahti, *Riv. Nuovo Cim.* **18**, 1 (1995).
13. J. V. Schwinger in *Quantum Theory of Angular Momentum*, ed. L. C. Biedernharn and H. Van Dom (Academic Press, New York, 1965).
14. H. Paul, P. Törmä, T. Kiss and I. Jex, *Phys. Rev. Lett.* **76**, 2464 (1996)
15. O. Steuernagel, *Opt. Comm.* **138**, to appear.
16. M. G. A. Paris, *Phys. Rev.* **A53**, 2658 (1996).
17. M. G. A. Paris, A. Chizhov and O. Steuernagel, *Opt. Comm.*, **134**, 117 (1997).
18. D. N. Klyshko, *Sov. Phys. JETP* **28**, 522 (1968); D. C. Burnham and D. L. Weinberg, *Phys. Rev. Lett.* **25**, 84 (1970).
19. S. Ya. Kilin and D. B. Horoshko, *Phys. Rev. Lett.* **74**, 5206 (1995).