Poincaré perturbation method[5] for solving (2); the results are in agreement with the previous ones, at least to linear terms in $\gamma$. For the sake of completeness, an outline of the Poisson-Poincaré procedure in the case where the glider moves downward is now given.

To use the Poisson-Poincaré method, write

$$v = v_0 + v_1 + v_2 + \cdots, \tag{13}$$

where the terms on the right-hand side represent the zeroth-, first-, second-, ··· order contributions in powers of the viscous damping coefficient $\gamma$. To obtain first-order results, one must now solve simultaneously

$$\frac{dv_0}{dt} = -a, \quad \frac{dv_1}{dt} = -\gamma v_0, \tag{14}$$

subject to the initial conditions given earlier. The solution is

$$v = -at + (1/2)\gamma a t^2 \tag{15}$$

to linear terms in $\gamma$, with

$$x = S_1 - (1/2)at^2 + (1/6)\gamma a t^3. \tag{16}$$

Next, square (15), omit the quadratic term in $\gamma$, and let $v = v_1$ where $x = 0$, at time $t = t_1$. As a result, (15) and (16) yield

$$v_1^2 = 2aS_1 - (2/3)\gamma a^2 (2S_1/a)^{3/2}. \tag{17}$$

This result is identical to that predicted by (8) or (12), at least to that order. A complete analysis will therefore yield (9), as already claimed.

## III. CONCLUSION

In summary, we have argued that undergraduate physics curricula should offer more advanced experimental programs in classical mechanics. We have given a simple case of a problem that could be assigned as a laboratory project at a level slightly above the usual introductory one. We have also presented a simple theory for the effect and shown by comparison with experiment that the model works and gives clear results. We think that this project would be feasible and interesting for undergraduates at the second year level of a four year program in physics.


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# Phase shift between the transmitted an the reflected optical fields of a semireflecting lossless mirror is $\pi/2$


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The aim of this note is to point out the existence of a general property of mirrors which, although important for the full understanding of optical interferometers, I have not found explicitly stated in any optics textbook.

Consider, for instance, a Michelson interferometer (see Fig. 1) illuminated by a linearly polarized plane wave with electric-field complex amplitude $E_o$. The electric field $E_t$ of the output beam is the superposition of the two contributions $E_t'$ and $E_t''$, originated, respectively, by the two paths *AHBHD* and *AHCHD*. The interferometer produces also a reflected field $E_r$ which is the superposition of the two contributions $E_r'$ and $E_r''$ following the two paths *AHBHA* and *AHCHA*. By calling $t = t_o \exp(i\phi_T)$ and $r = r_o \exp(i\phi_R)$ the field transmission and reflection coefficients of the beam splitter, and $\phi_1$ and $\phi_2$ the phase shifts due to reflection on mirrors *B* and *C*, the following expressions are obtained for the phases $\phi_t'$, $\phi_t''$, $\phi_r'$, $\phi_r''$ of the four fields $E_t'$, $E_t''$, $E_r'$, $E_r''$:

$$\begin{aligned} \phi_t' &= \frac{4\pi}{\lambda} d_1 + \phi_T + \phi_R + \phi_1, \\ \phi_t'' &= \frac{4\pi}{\lambda} d_2 + \phi_T + \phi_R + \phi_2, \\ \phi_r' &= \frac{4\pi}{\lambda} d_1 + 2\phi_T + \phi_1, \\ \phi_r'' &= \frac{4\pi}{\lambda} d_2 + 2\phi_R + \phi_2. \end{aligned} \tag{1}$$

By assuming that no absorption is present in the interferometer (lossless beam splitter), the energy conservation condition $|E_o|^2 = |E_t|^2 + |E_r|^2$ gives

$$\phi_t' - \phi_t'' = \phi_r' - \phi_r'' + \pi. \tag{2}$$

By substitution of Eqs. (1) into (2), the following relation is derived

$$\phi_R = \phi_T + \pi/2. \tag{3}$$

Note that Eq. (3) is derived without making any assumption on the structure of the semireflecting lossless mirror *H* which may consist of an arbitrary number of dielectric

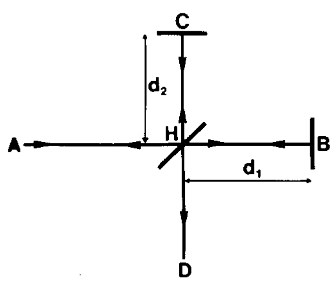

Fig. 1. Scheme of the Michelson interferometer. The electric field vectors vibrate perpendicularly to the plane of the drawing. *A* and *B* are totally reflecting mirrors, *H* is a semireflecting mirror.



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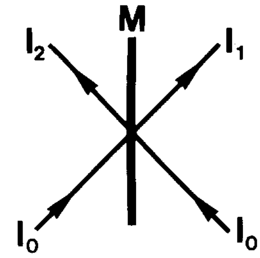


Fig. 2. Semireflecting mirror illuminated symmetrically by two beams having the same intensity $I_0$.

layers on a glass plate.

An even simpler demonstration of Eq. (3) can be obtained by considering the situation depicted in Fig. 2, where two beams having electric fields of equal amplitude and phase are impinging on a semireflecting mirror from two opposite sides, with the same incidence angle. If the mirror is lossless, one should expect for symmetry reasons that $I_1 = I_2 = I_o$. Now $I_1 = |E_1|^2$, where $E_1$ is the sum of the two contributions due, respectively, to the reflection of the right-hand side beam and to the transmission of the left-hand side beam. Hence $E_1 = rE_o + tE_o$, and $I_1 = |t + r|^2 I_o$. The condition $I_1 = I_o$ gives $|t + r|^2 = 1$, that is

$$tt^+ + rr^+ + 2\mathrm{Re}\,(tr^+) = 1.$$

Since $tt^+ + rr^+ = 1$, the real part of the product $tr^+$ is equal to 0, that is $\cos(\phi_R - \phi_T) = 0$, or $\phi_R - \phi_T = \pi/2$. An interesting feature of this latter proof is that it shows Eq. (3) to be valid independently of the angle of incidence.

The case of lossy mirrors does not give rise to simple general relations such as Eq. (3), and the phase difference $\phi_R - \phi_T$ can depart substantially from $\pi/2$, as shown, for instance, by the interferometric experiment described in Ref. 1 where a half-silvered plate is used as a beam splitter.

Thanks are due to M. Giglio, A. Sona, and F. Amman, for useful discussions.

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## Aristotle and the increasing weight of falling bodies

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Recently in this journal, our attention has twice been drawn to the following passage from Aristotle:

> . . . earth moves more quickly the nearer it is to the center and fire the nearer it is to the upper place. But if movement were infinite, speed would be infinite also; and if speed then weight and lightness. For as superior speed in downward movement implies superior weight, so infinite increase of weight necessitates infinite increase of speed.

Commenting on this passage, Barry M. Casper wrote in his article[1]:

> . . . here Aristotle seems to be saying something which sounds quite strange: as an object falls it increases its speed and "superior speed in downward movement implies superior weight." In other words, a falling object increases its weight as it falls!

Commenting on Casper's article, Redding cited the above passage as an example of a passage whose meaning is difficult for us to be sure of.[2]

I would not disagree with either of these two authors that this passage sounds peculiar. In it Aristotle does say something which, no doubt, most of us would not have expected him to have said. In this note I would like to point out, however, that there is a relatively straightforward explanation of why Aristotle would be led to assert that the weight of an object increases as it falls.

To begin with, it is worth emphasizing that Aristotle did observe that the speed of an object falling in air increases. That some of us may not previously have been aware of this is not surprising. Most of the time Aristotle does give the impression that he thought that bodies fell at a constant speed. But as Casper was careful to note in his article, there are passages that reveal Aristotle's awareness of the acceleration of falling bodies.

It is also worth emphasizing that Aristotle's physics did not include any action-at-a-distance theories. Specifically, in the case of falling bodies, Aristotle had no conception of an external force of gravity as the cause of motion. Instead, Aristotle thought of the cause as being internal to the body itself. He thought that a body fell as the result of a natural tendency on its part to move toward the center of the universe, and he thought of its weight as a measure of the strength of this natural tendency.

Keeping the preceding ideas in mind and recalling that Aristotle's was a noninertial physics, we are now in a position to understand why Aristotle would be led to assert that the weight of a falling object increases during its fall. In Aristotle's physics a constant force was said to produce a constant speed. For instance, the motion of a cart moving at a constant speed was explained by the continuing action of a constant force. A decrease in the speed of the cart could be explained by a decrease in the force; an increase in speed by an increase in force. The situation that Aristotle encountered in the case of falling bodies corresponds to the case in which the cart moves with increasing speed. Just as the increasing speed of the cart can be explained by an increase in the force that is causing its motion, so the increasing speed of a falling body can be explained by an increase in the force that is causing its motion. In the case of a falling body, the force that is causing its motion is its natural tendency to fall. Since the weight of the body is the

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