Energy exchange between free electrons and light in vacuum

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Momentum is exchanged between free electrons and light when the interaction length is limited. An analysis and a computer simulation determine the characteristics of this energy transfer. Possible experimental schemes at optical and infrared wavelengths are explored.

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I. INTRODUCTION

There are applications in which one is interested in coupling visible or infrared electromagnetic waves to a free-electron beam: to obtain radiation from the particle beam or to accelerate the particles. Several approaches that provide this coupling are the free-electron laser,[1] the Smith-Purcell effect,[2] and inverse Čerenkov radiation.[3] Each of these techniques involves retardation of the phase velocity of the wave, or a Fourier component of the wave, so that wave-vector matching can be achieved.

Alternatively, energy exchange in vacuum can occur by limiting the interaction length so that the phase slippage of the particle in the field is restricted to less than $\frac{1}{2}\pi$. For relativistic electrons this interaction length can be many radiation wavelengths long. In a previous paper[4] an analysis was presented assuming the field to be a plane wave, infinite in extent in the direction of propagation, and with a Gaussian variation in intensity in the transverse direction. It was indicated that this assumed field did not satisfy Maxwell's equations, and therefore a better characterization of the field is required.

In the present paper the field is represented by a summation of plane waves propagating in slightly different directions. Thus, the radiation can be Fourier synthesized with components each of which satisfy Maxwell's equations. In contrast with the result of the previous paper[4] it is found that the net energy exchange between a particle and a focused light beam is zero if the interaction region is unlimited. This result is an immediate consequence of having Fourier components that are plane waves. However, by terminating the field, either by reflection, absorption, or diffraction, a net energy exchange can occur.

II. ANALYSIS

As shown in Fig. 1, it is assumed that a focused Gaussian laser beam is propagating in the z direction, polarized in the x direction at $z = 0$, and uniform in the y direction. An electron moving in the x-z plane intersects the z axis at an angle ψ.

The electromagnetic field components may be represented by a superposition of plane waves traveling in different directions,

$$E_x(x,z,t) = \exp(-i\omega t)\int_{-\pi/2}^{\pi/2} d\theta\, a(\theta)\cos\theta \times \exp(ikx\sin\theta + ikz\cos\theta),$$

$$E_z(x,z,t) = -\exp(-i\omega t)\int_{-\pi/2}^{\pi/2} d\theta\, a(\theta)\sin\theta \times \exp(ikx\sin\theta + ikz\cos\theta), \qquad (1)$$

where ω is the frequency, $k = 2\pi/\lambda$, and λ is the wavelength. This angular spectrum is the Fourier transform of the transverse spatial distribution of the laser light.

Assuming a Gaussian-shaped spot located at $z = 0$,

$$E_x(x,0,t) = E_0 \exp(-i\omega t)\exp(-x^2/w^2), \qquad (2)$$

where w is the beam spot size and $E_0 = |E_0|\exp(i\phi)$ is the complex field amplitude, the expression for $a(\theta)$ becomes

$$a(\theta) = \frac{kw}{2\pi^{1/2}} E_0 \exp\left[-\left(\frac{wk\sin\theta}{2}\right)^2\right]. \qquad (3)$$

The expansion, as given by Eqs. (1) and (3), closely represents the field if $2\pi w/\lambda \gg 1$, which is satisfied for the considered range of parameter values.

The change in particle momentum in the direction parallel to its initial motion is given by the force equation

$$\frac{dp_\parallel}{dt} = -qE_x\sin\psi + qE_z\cos\psi, \qquad (4)$$

where q is the charge of the particle and $p_\parallel$ is the momentum change produced by the field along the direction of initial motion. For momentum changes small compared to the initial momentum, the energy change ΔW is given by

$$\Delta W \simeq \beta c\, p_\parallel, \qquad (5)$$

where c is the velocity of light and β is the ratio of particle velocity to the velocity of light.

To determine ΔW the Born approximation is used, i.e., the particle trajectory in the interaction region is assumed to be unaltered by the presence of the field. Taking $z = 0$ at $t = 0$, we have

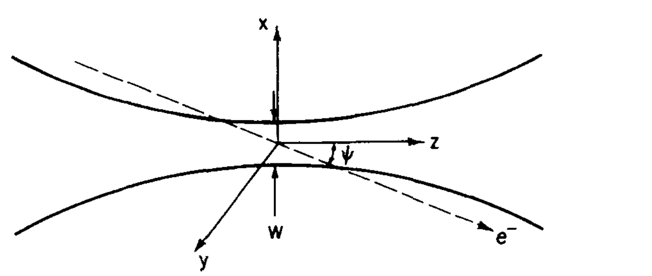

FIG. 1. Electrons move at an angle ψ relative to the z axis through a focused light beam that has a Gaussian variation in the x direction and is uniform in the y direction. The field is polarized in the x direction at $z = 0$.

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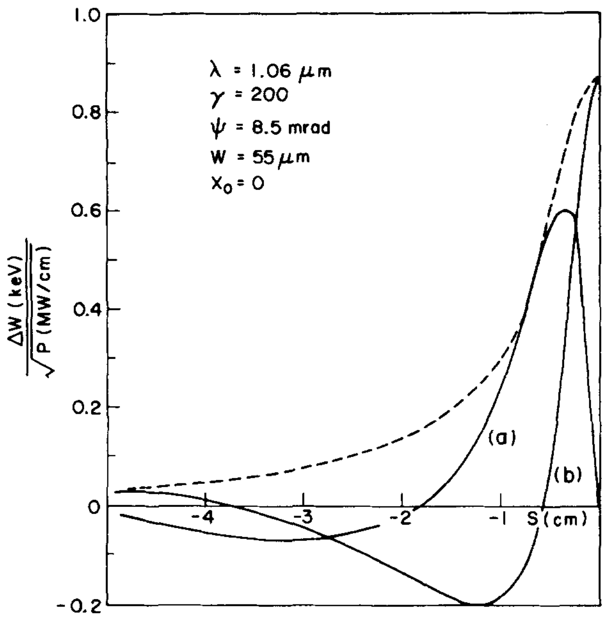

FIG. 2. The ratio $\Delta W/P^{1/2}$ as a function of distance along the direction of electron motion, where ΔW is the energy exchange and P is the power per unit length in the y direction in the laser beam. For curve (a) $\phi = 0$ and for curve (b) $\phi = \frac{1}{2}\pi$ where ϕ is the phase constant that appears in Eq. (8). The dashed line is the envelope of the curves for different phase constants and therefore is the maximum value of $\Delta W/P^{1/2}$ at any given position.

$$z = \beta ct \cos\psi, \tag{6}$$

$$x = x_0 - \beta ct \sin\psi, \tag{7}$$

where x_0 is the x position of the particle at $z = t = 0$. The simultaneous solution of Eqs. (1)–(7) gives the expression for ΔW

$$\Delta W = \frac{-qw|E_0|}{2\pi^{1/2}} \int_{-1}^{1} du \exp(-\alpha^2 u^2) \frac{d\{\ln[f(u)]\}}{du}$$

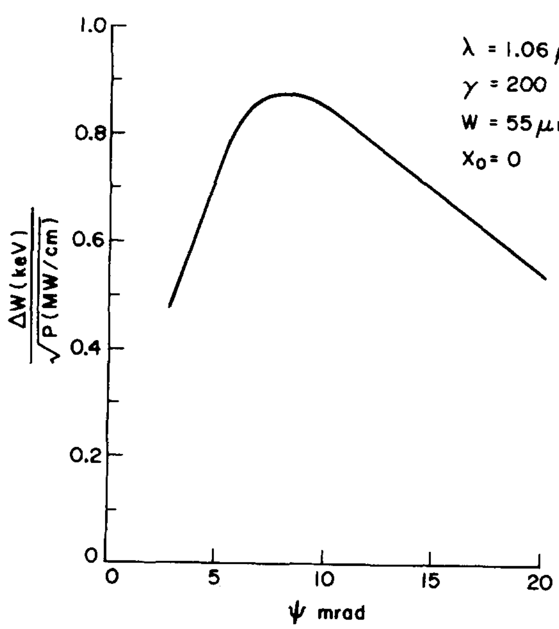

FIG. 3. The maximum value of $\Delta W/P^{1/2}$ as a function of the angle ψ between the electron velocity and the light wave vector.

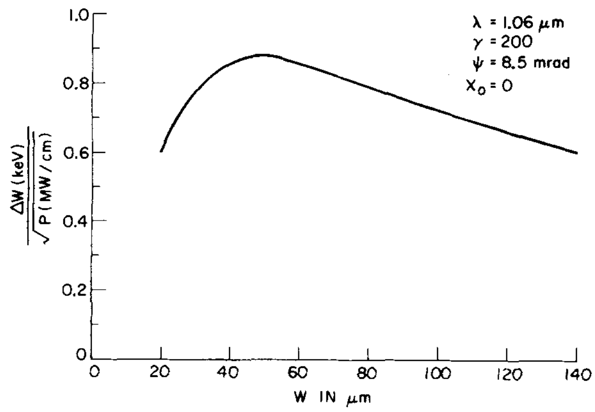

FIG. 4. The maximum value of $\Delta W/P^{1/2}$ as a function of the laser beam spot size.

$$\times \sin\left(\frac{2\pi s}{\beta\lambda} f(u) - \frac{2\pi x_0}{\lambda} u - \phi\right), \tag{8}$$

where $\alpha = \pi w/\lambda$, $s = z/\cos\psi$ is the distance measured along the direction of particle motion, and

$$f(u) = 1 + \beta u \sin\psi - \beta(1 - u^2)^{1/2} \cos\psi. \tag{9}$$

III. RESULTS

Equation (8) has been integrated numerically to determine ΔW for various sets of parameters. Figure 2 shows the ratio $\Delta W/P^{1/2}$, where P is the laser power per unit length in the y direction, as a function of the distance s along the direction of electron travel. Curve (a) is $\Delta W/P^{1/2}$ for phase $\phi = 0$, and curve (b) is for $\phi = \frac{1}{2}\pi$. Curve (a) is an odd function of s and curve (b) is an even function of s. The dashed line is the envelope of these curves and therefore shows the maximum $\Delta W/P^{1/2}$ for any given value of s. Input parameters for Fig. 2 are the laser wavelength $\lambda = 1.06\,\mu$m and the particle energy $\gamma = (1 - \beta^2)^{-1/2} = 200$. The angle of intersection $\psi = 8.5$ mrad, the laser light spot size $w = 55\,\mu$m, and $x_0 = 0$ were chosen to maximize $\Delta W/P^{1/2}$. It is seen from Fig. 2 that, in contrast with the plane wave approximation,[4] if the interaction length is unlimited, $\Delta W = 0$.

Figure 3 illustrates the maximum energy exchange (i.e., the maximum value of the dashed curve in Fig. 2) as a function of the angle ψ between the electron velocity and the light wave vector. The ψ corresponding to maximum ΔW is longer than the value $\psi = \gamma^{-1}$ predicted from the plane wave assumption.

Figure 4 shows the maximum energy exchange as a function of the laser beam spot size w. The plane wave assumption gives an optimum w that is $w = \lambda\gamma/2\pi = 33.7\,\mu$m, which is smaller than the optimum w in Fig. 4.

The effect of displacement of the electron along the x direction, so that it misses the center of the light beam focus, is illustrated in Fig. 5. Numbers written adjacent to points on the electron trajectories give the maximum values for $\Delta W/P^{1/2}$ at the corresponding positions. It is seen that ΔW falls off fairly rapidly at $z = 0$ as the electron is displaced in x, but the falloff is much slower along a path close to the z axis. The values of $\Delta W/P^{1/2}$ shown near the z axis are at the position of maximum $\Delta W/P^{1/2}$.

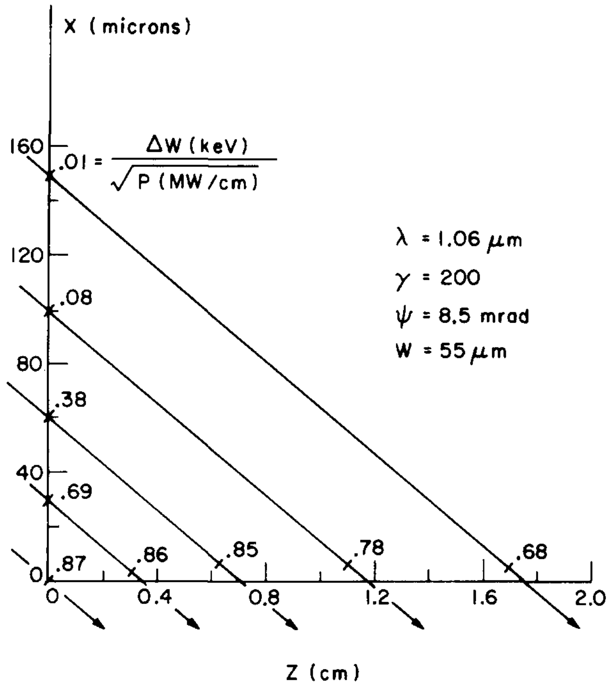

FIG. 5. The effect of displacement of the electron along the x direction. Numbers written adjacent to points on the electron trajectories give the maximum values for $\Delta W/P^{1/2}$ at the corresponding positions.

A calculation has been performed without the Born approximation, allowing the particle trajectory to be altered by the electromagnetic field. The approximation becomes invalid when the laser power is sufficiently high so that an electron receiving maximum ΔW gains about one-quarter of a wavelength within the interaction region on an electron receiving zero ΔW. For the parameter values given in Fig. 2, this requires an energy change of 10 MeV or greater. As discussed in Sec. IV, the energy exchange of interest is on the order of 20 keV, in which case the exact solution for ΔW differs from the Born approximation solution by one part in 10^4.

A light beam focused in two dimensions has also been considered, in which the field has a Gaussian variation in the x and y directions. (Refer to Fig. 1.) The energy exchange ΔW for a particle passing through the focus is almost the same for the two-dimensional and one-dimensional focused light beams if the axial electric field is the same in both cases. This means that the power required to obtain a specified ΔW for a one-dimensional beam must be $\simeq w_x/w_y$ times the power in a two-dimensional beam, where w_x is the spot size along the x direction and w_y is the spot size along the y direction. The advantage to using a one-dimensional light beam is that the y dimension of the particle beam can be increased, thereby allowing for a greater electron current.

IV. DISCUSSION

One method for obtaining energy exchange between focused laser light and free electrons over a limited interaction length is to terminate the electromagnetic field in the region of intense field strength. Since the field strengths of interest are on the order of 10^6 V/cm or larger, most materials will be damaged or vaporized.

A possible technique for terminating the high fields of a pulsed laser is to use a moving thin foil so as to present a new surface for each pulse. For a foil that is 10 μm thick and with a field strength of $\simeq 10^6$ V/cm, the burn-through time for light is in excess of 8 nsec[5] so that this does provide a means for stopping the radiation.

If the purpose of the energy exchange is to produce bunching of the particles so that the electrons can reradiate in the manner of a klystron, then the energy ΔW must exceed the random energy of the particle beam.[6] There will be two sources of random energy: that which is intrinsic to the incident particle beam, and that which is introduced by the foil used to terminate the light. For the superconducting accelerator at Stanford the intrinsic energy spread at $\gamma = 200$ is $\simeq$20 keV, and a 20-μm Al foil introduces $\simeq$10-keV random energy. This means that the total energy spread is $\simeq$22 keV. From Fig. 2 it is seen that the laser power per unit length must be > 640 MW/cm for significant bunching to occur, and this power requirement is well within the capability of pulsed Nd lasers. The corresponding field strength at the focus is 6.0×10^6 V/cm.

The alignment of the foil is important for this determines the amount of energy exchange for particles that do not pass through the center of the light-beam focus (refer to Fig. 5) and also affects the thickness of the foil presented to the electrons. To minimize the random energy on the particle beam the thickness should be as small as possible, which suggests that the foil should be normal to the electron velocity. On the other hand, from Fig. 5 it is seen that the foil should be nearly parallel to the particle direction so as to maximize ΔW for particles that do not pass through the focus. There will be an optimum foil position, between the normal and parallel positions, which will depend upon the x dimension of the electron beam.

The foil can be eliminated by altering the electron's trajectory as it passes through the interaction region. From Figs. 2 and 3 we see that if the direction of the particle is changed by $\simeq$8 mrad in a distance less than the distance over which most of the energy exchange occurs (i.e., less than 1 cm), this is equivalent to an abrupt termination of the field. For example, with transverse magnetic field of 10 kG over a distance of 0.5 cm, and using the parameter values given in Figs. 2 and 3, the electron retains 30% of its peak energy exchange without the use of a foil.

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