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# LETTERS TO NATURE

## A single quantum cannot be cloned

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**If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.**

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6–8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \tag{1}$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle \tag{2}$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \tag{3}$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha=\beta=2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle) \rightarrow \alpha|A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle+\beta|A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \tag{4}$$

If the apparatus states $|A_{\text{vert}}\rangle$ and $|A_{\text{hor}}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle+\beta|\leftrightarrow\leftrightarrow\rangle \tag{5}$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^{+}_{\text{vert}}+\beta a^{+}_{\text{hor}})^2|0\rangle=\alpha^2|\updownarrow\updownarrow\rangle+2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle+\beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

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Milonni (unpublished work) has shown that the process of stimulated emission does not lead to quantum amplification, because if there is stimulated emission there must also be—with equal probability in the case of one incoming photon—spontaneous emission, and the polarization of a spontaneously emitted photon is entirely independent of the polarization of the original.

It is conceivable that a more sophisticated amplifying apparatus could get around Milonni's argument. We have therefore presented the above simple argument, based on the linearity of quantum mechanics, to show that no apparatus, however complicated, can amplify an arbitrary polarization.

We stress that the question of replicating individual photons is of practical interest. It is obviously closely related to the quantum limits on the noise in amplifiers[10,11]. Moreover, an experiment devised to establish the extent to which polarization of single photons can be replicated through the process of stimulated emission is under way (A. Gozzini, personal communication; and see ref. 12). The quantum mechanical prediction is quite definite; for each perfect clone there is also a randomly polarized, spontaneously emitted, photon.

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# The Crab Nebula's progenitor

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**The study of supernovae is hampered by an insufficient knowledge of the initial stellar mass for individual supernova. Because of large uncertainties in estimating both the total mass of a remnant (including the pulsar or black hole) and any mass loss during the pre-supernova stages, the main sequence mass of the progenitor cannot be accurately determined from observations alone. To calculate an initial mass, one must rely on a combination of both theory and observation. Limits on the progenitor's mass range can be estimated by the presence of a compact remnant and comparison of the observed nebular chemical abundances with detailed evolutionary calculations[1]. The Crab Nebula is an excellent choice for investigation because it contains a unique combination of characteristics: a central neutron star (pulsar) and a bright, well studied nebula having little or no swept-up interstellar material. In fact, several studies[1–4] have suggested an initial mass of $\sim 10\,M_\odot$ for the Crab progenitor. Recently, Davidson *et al.*[5], quoting two of us (K.N. and W.M.S.), state that the Crab's progenitor had a mass slightly larger than $8\,M_\odot$. Here we present in detail the reasoning behind this statement and suggest the explosion mechanism.**

Briefly, the Crab consists of a pulsar (assumed here to have a mass of $\approx 1.4\,M_\odot$) and a nebula mass of $1.2$–$3.0\,M_\odot$ (refs 5, 6) which has a helium overabundance of $1.6 < X_{\rm He}/X_{\rm H} < 8$ (where $X$ is an element's mass fraction). The oxygen abundance ($X_{\rm O}$) is $\sim 0.003$ (refs 5, 6), which is less than the solar value of 0.007, while the oxygen-to-hydrogen ratio is approximately solar. The carbon-to-oxygen ratio is $0.4 < X_{\rm C}/X_{\rm O} < 1.1$ (ref. 5). Nitrogen may be slightly overabundant, while neon, sulphur and iron abundances are uncertain but are probably not greatly over- or underabundant. Because the Crab Nebula is helium-rich but not oxygen-rich, the hydrogen-rich (solar abundances) envelope and the helium layer of the progenitor star were ejected but the oxygen-rich layer below the helium layer was not. The lower layers must have formed the neutron star. The lower limit of the large helium-to-hydrogen ratio means that at least half of the ejected material must have come from the helium layer.

Arnett (ref. 7 and refs therein) systematically evolved helium cores of various masses ($M_\alpha$) into late stages of evolution. He[1] compared Davidson's[8] derived abundances of the Crab nebula with calculated abundances from the $M_\alpha = 4.0\,M_\odot$ model, which was his lowest-massed, highly evolved helium core (corresponding to approximately a $15\,M_\odot$ star). Combining all the material above the helium-burning shell (his case B) with enough interstellar material to obtain $X_{\rm He}/X_{\rm H} = 8$, he found good agreement with $X_{\rm N}/X_{\rm He}$ and $X_{\rm O}/X_{\rm He}$ of Davidson's[8] 'model 1'. However, the calculated value of $X_{\rm C}/X_{\rm He}$ was too large by a factor of 30[illegible] At that time, the Crab's carbon abundance had not been dire[illegible] measured and Arnett suggested several possibilities: the inferred carbon abundance was too low, the carbon was hidden in the filaments, or a lower-mass helium core, $\sim 3\,M_\odot$, was more appropriate.

Using recent UV observations with the International Ultraviolet Explorer, Davidson *et al.*[5] have established that the carbon abundance is nearly solar. They also showed that the hydrogen and helium seemed to be fairly well mixed and, as carbon is convectively mixed in the helium layer, this would argue against carbon being hidden in the filaments. However, IR observations by Dennefeld and Andrillat[9] showed that the strength of [C I] λ9,850 relative to [S III] λ9,069 varied with position in the Crab. The strongest [C I] line would indicate a rather large carbon abundance if the ionizing flux is constant. Whether the IR observations indicate variation in the carbon abundance, variation in the ionizing flux, or high densities in neutral cores is not known. For the remainder of this report we will assume the carbon abundance as determined by Davidson *et al.*[5].

The existence of a pulsar in the Crab indicates that the progenitor's mass was larger than the upper mass limit ($8 \pm 1\,M_\odot$)[10] for degenerate carbon ignition. Degenerate carbon ignition results in carbon deflagration[11] which completely disrupts the star, leaving no compact remnant. Lower-mass stars that lose enough mass to avoid degenerate carbon burning eventually become white dwarfs. Stars massive enough ($\geq 8\,M_\odot$) to burn carbon non-degenerately will eventually undergo a core collapse initiated either by electron capture[12] onto Mg, Ne and O or by burn-out of all the available fuel[13,14]. When the collapsing core reaches neutron-star densities, stability is regaine[illegible] Although detailed calculations of the collapse remain inconcl[illegible]ive, it is generally felt that the core will overshoot its equilibrium position and then rebound, initiating a shock wave[4]. This shock wave ejects the outer material but not the core, resulting in both a supernova nebula and a pulsar[15]. In more massive stars

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