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Analytical dynamics

Numismatic gyrations

The familiar shuddering motions of spinning coins as they come to rest are not at all intuitive. Moffatt's analysis (*Nature* **404,** 833–834; 2000) identifies air viscosity as the causative factor in coin jitter, so we tested this hypothesis by studying coins spinning in a vacuum. We discovered that the presence of air has little effect on the final motions of the coins, indicating that slippage and friction between the coin's edge and the supporting surface might cause the vibrations that accompany the end of the spin.

Casual observation of various objects spun on a tabletop indicates that compression of trapped air does not qualitatively affect the complex motions of spinning disks. We noted that a ring-shaped bell-jar lid, a short cylinder or the lid of a shoe-polish can — tested with either the rim or the flat side down — show a comparable behaviour: they spin on edge, topple over, then wobble to a shuddering halt. The universality of this motion is surprising in light of the air-viscosity mechanism proposed by Moffatt. As rings do not trap air the way solid disks do, these objects should generate shear forces of different magnitudes. The similar kinetic behaviour of these objects appears to contradict a decisive role for air viscosity.

The Dutch 2.5-guilder coin has magnetic properties that allow it to be spun with a precise frequency on a magnetic stirrer. We placed the coin in a glass desiccator that had a slightly concave bottom, brought it to a spin of approximately 10 Hz, and observed the motions of the slowing coin after the desiccator was lifted carefully from the stirring platform. The desiccator could be evacuated to less than 1 mtorr of air pressure.

Coins *in vacuo* spun on average for 12.5 s; coins in air spun on average for 10.5 s (average of 10 observations each). This difference in time can be attributed to a difference in the time the coin was spinning upright on its edge. The time from the onset of tumbling to standstill did not differ markedly and was about 4 s under both conditions. With or without air, the coin displayed the same characteristic final motions. We conclude that the presence or absence of air may have some effect on the upright duration of the spin, but has little effect on the final whirling motions that bring coins to rest. In contrast, Moffatt's analysis would predict a very long wobbling time for a coin in a vacuum.

We propose an alternative explanation for the jerking motions with which coins lose their spin. A coin toppling from rotation on edge preserves its rotational energy so that the axis of rotation changes from the plane of the coin to one perpendicular to the coin. The coin now must wobble on its edge. As Moffatt indicates, the friction is minimal when the point of contact between the supporting surface and the wobbling coin describes a circle with radius $R\cos(\alpha)$ (see his Fig. 1). But the coin is not free to choose any rotation speed. The gravitational force supplies a moment that interacts with the spin moment and the wobble moment. As a result, the coin is subject to precessing forces that rub the coin's edge in a jerking motion against the tabletop. We believe that this sliding friction temporarily lifts the coin, moving the point of contact between edge and supporting surface in a rapid staccato. It is this friction that brings the coin to a final rest.

The role of surface friction can be readily confirmed with the toy that inspired Moffatt's analysis. When placed on a table rather than on its slippery platform, Euler's disk rapidly comes to rest, illustrating the influence of the roughness of the supporting surface on the spinning time. Air viscosity may play a role in stopping 'theoretical' coins. Real-world coins, thrown on a table, do not need a finite-time singularity to control their spin. Edges rubbing against the tabletop explain the rapid dissipation of monetary momentum.

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Moffatt replies — It is true that there are a number of possible dissipative mechanisms for the rolling disk in addition to viscous dissipation in the surrounding air: vibration of the supporting surface, rolling friction due to plastic deformation at the point of rolling contact, and, as suggested by van den Engh *et al.*, dissipation due to slipping rather than rolling. The 'adiabatic' equation that I used, relating the precessional angular velocity Ω to the angle α, is valid only under the rolling condition, and experiments indicate that this condition is indeed satisfied for the 'toy' Euler's disk rolling on a flat, smooth horizontal glass plate placed on a firm table (V. A. Vladimirov, personal communication). I believe therefore that slipping does not occur in this case.

The problem really is to identify the dominant dissipative mechanism, for a given disk and a given surface, and then to evaluate the associated rate of dissipation of energy as a function of the angle α (which is proportional to the energy). If this rate of dissipation of energy turns out to be proportional to a power of α, where the exponent of this power, λ say, is less than one, then, under the adiabatic approximation, a finite-time singularity (for which Ω becomes infinite) will occur.

The air-viscosity mechanism I described yields $\lambda = -2$ (note that air viscosity is relatively insensitive to pressure, so that partial evacuation of the vessel in which the disk experiment is conducted should have only a small effect). An improved theory that takes account of oscillatory Stokes layers on the disk and supporting surface (L. Bildsten, personal communication) yields $\lambda = -5/4$. If 'rolling' friction is assumed to dissipate energy at a rate proportional to Ω, then $\lambda = -1/2$. Careful experiments under a variety of conditions should distinguish between these various possibilities.

I chose to focus on viscous dissipation because that is the only mechanism for which a fundamental (rather than empirical) description is available, namely that based on the Navier–Stokes equations of fluid dynamics. The fact that the air-viscosity mechanism exhibits the strongest singularity as α tends to zero suggests that this mechanism will always dominate when α is sufficiently small. For larger α and smaller disks (such as the 2.5-guilder coin), rolling friction is an equally plausible candidate (A. Ruina, personal communication), but determination of the associated rate of dissipation of energy (in terms of the physical properties of the disk and the surface) involves solution of the equations of (possibly plastic) deformation in both solids at the moving point of rolling contact, a difficult problem, which, so far as I am aware, still awaits definitive analysis.

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