# Higher Order QED Effects

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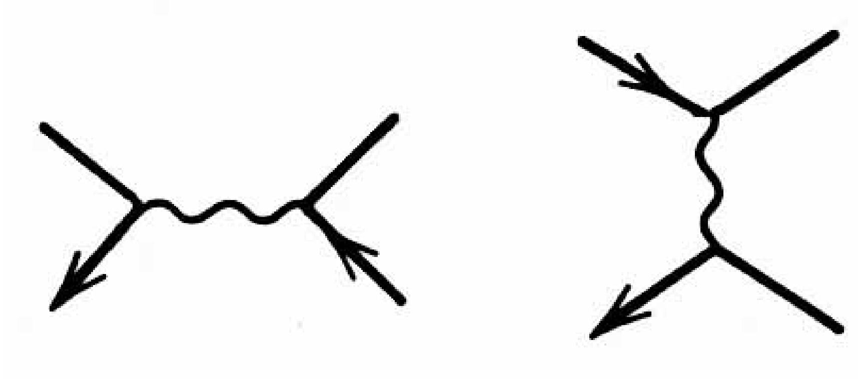
"...in 30 minutes (including discussion) review exhaustively the important aspects of the subject, with emphasis on the most recent experimental results, and their implications."

http://puhep1.princeton.edu/~mcdonald/e144/qedtrans.ps

## What is Higher Order QED?

LANL preprint server: Exactly 1 paper found for "higher order QED" (but $\approx$ 750 found for "QED").

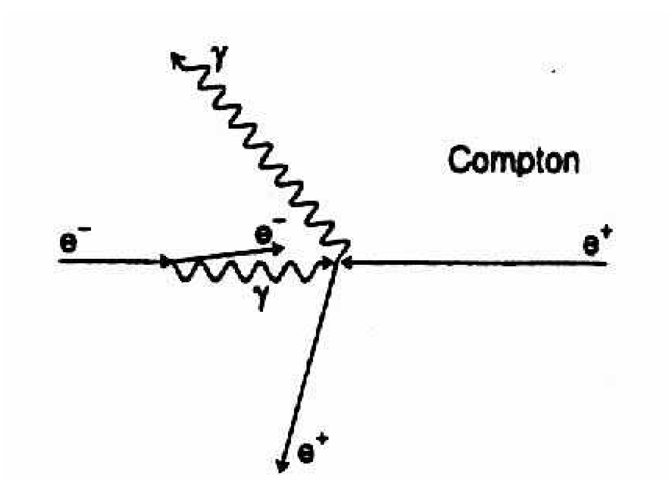
What is low order QED?

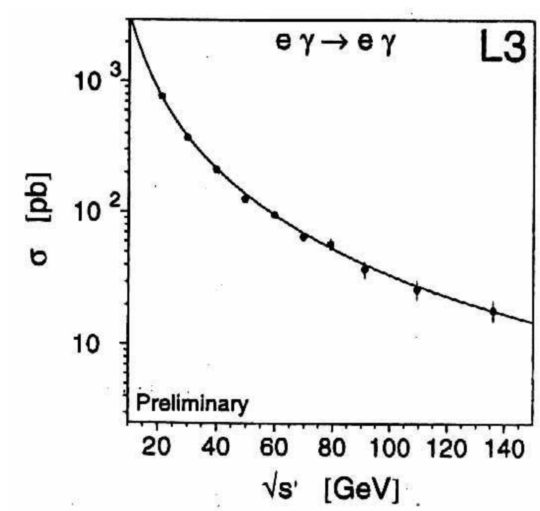


Higher Order QED is anything else?

*I.e.*, Any process with more than one photon?

Thus, recent "discovery" at LEP of Compton scattering:



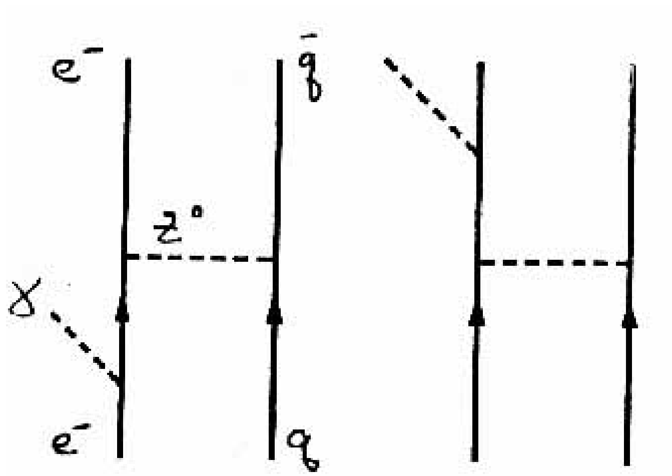
# Trees

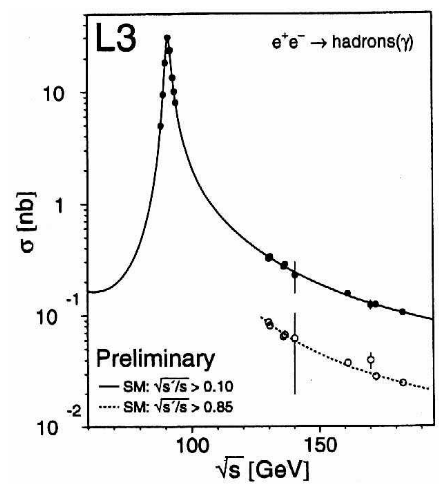
More usual definition: Count vertices;

Higher order $\Leftrightarrow$ More than 2.

[Classical: dipole radiation is lowest order.]

Simplest higher order: Trees (no loops) $\Rightarrow$ Radiative corrections.

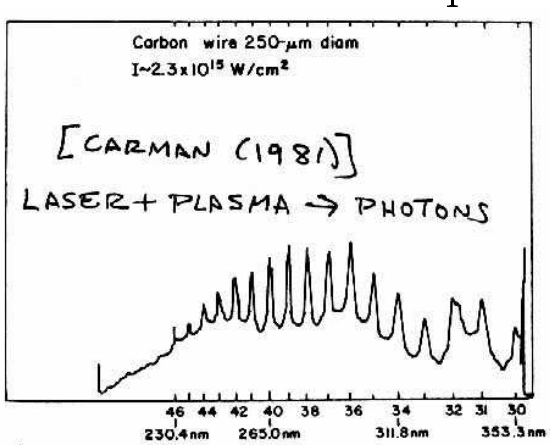


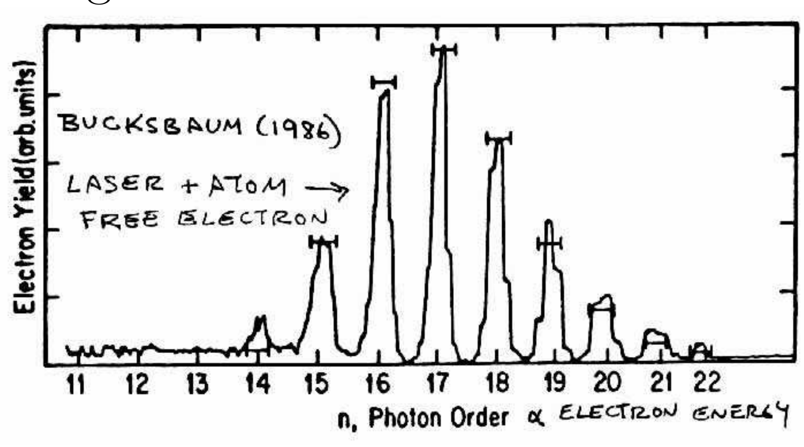




Perturbation series: $n$ vertices $\Rightarrow$ Rate $\propto \alpha^n$,

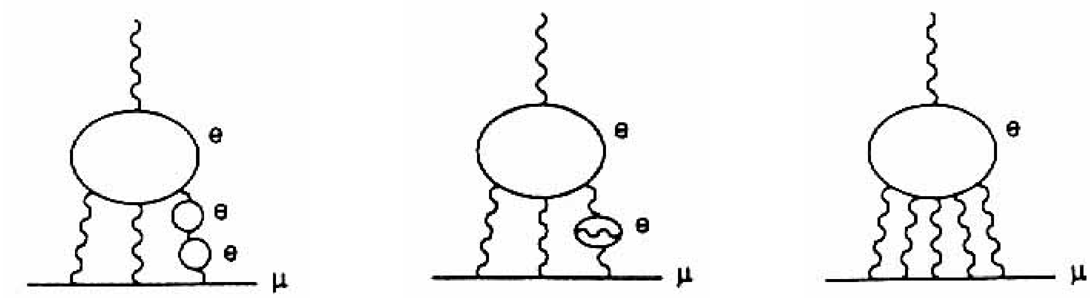
$\alpha = e^2/\hbar c = 1/137 \Rightarrow$ Higher order typically smaller.

But there is a nonperturbative regime...



## Loops




Calculations now for 10 extra vertices $\Leftrightarrow$ relative $\mathcal{O}(\alpha^5)$.

Four classic tests: [Reviews: Kinoshita (1990), Escribano (1997)]

- Hydrogen Lamb Shift: $\sigma_{\Delta E}(2S_{1/2} - 2P_{1/2}) = 2$ ppm

  [Theory limited by uncertainty in proton radius].

- Muonium hyperfine splitting: Expt. $-$ Theory $\approx 0.25$ ppm

  [muon mass, $\mathcal{O}(\alpha^3)$ terms, hadronic (+ weak) loops].

  New LAMPF data being analyzed; error $\rightarrow 0.1$ ppm.

- $e$ anomalous magnetic moment: Expt. $-$ Theory $\approx 25$ ppb

  [$\alpha$, $\mathcal{O}(\alpha^5)$ terms].

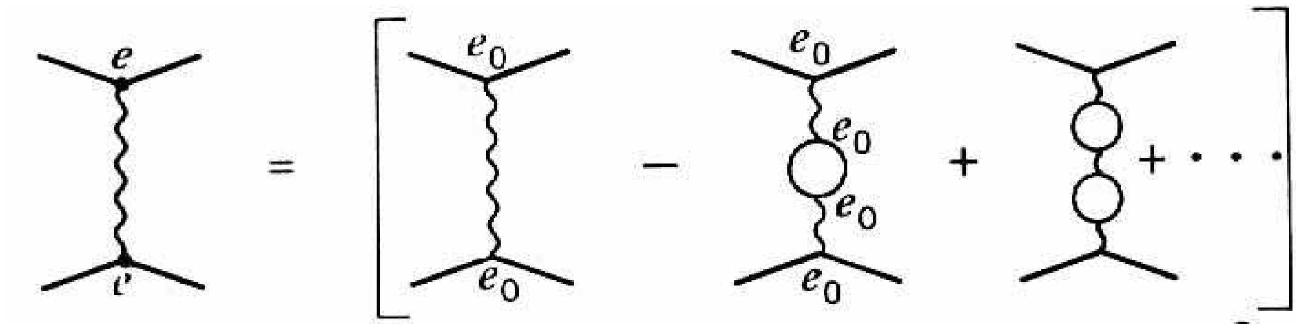
- $\mu$ anomalous magnetic moment: Expt. $-$ Theory $\approx 10$ ppm

  [$\mathcal{O}(\alpha^5)$ terms, hadronic (+ weak) loops].

  New BNL expt. starts in 2 months; error $\rightarrow 0.5$ ppm.

Trouble spot: Observed orthopositronium decay rate differs from theory by 6 $\sigma$; but theory is incomplete at relative $\mathcal{O}(\alpha^2)$.

# Running Coupling Constant




$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi}\ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

Extrapolation:

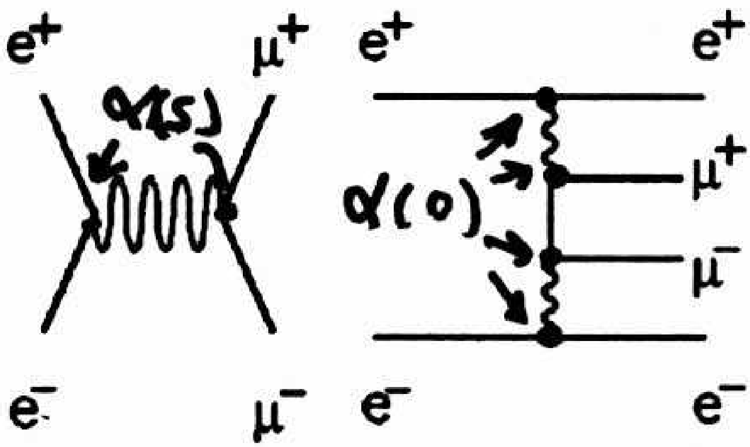
$\alpha^{-1}(M_Z^2) = 128.93 \pm 0.02;$

half of change due to hadronic corrections [Davier & Höcker, 1998]

TOPAZ result [Levine *et al.*, 1997]:

$\alpha^{-1}(Q^2 = (57.77\ \mathrm{GeV}/c)^2) = 128.5 \pm 1.8$ [Theory = 129.6].

Obtained by comparing $e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to e^+e^-\mu^+\mu^-$.



## Landau Pole Problem

For large $Q^2$, $\alpha$ grows arbitrarily large.

Can avoid by chiral symmetry breaking [Göckeler *et al.*, 1998].

## QED Phase Transition at Strong Coupling?

Suggested in lattice calculations [Kogut *et al.*, 1984 on].

$\Rightarrow$ New types of QED bound states.

## QED Phase Transition in Strong Fields?

$E > E_{\text{crit}} = m^2c^3/e\hbar = 1.3 \times 10^{16}$ V/cm = QED critical field,

above which spontaneous pair creation occurs.

No theory of strong field phase change.

"Evidence" of positron peaks in low-energy heavy ion collisions [Darmstadt] now largely withdrawn.

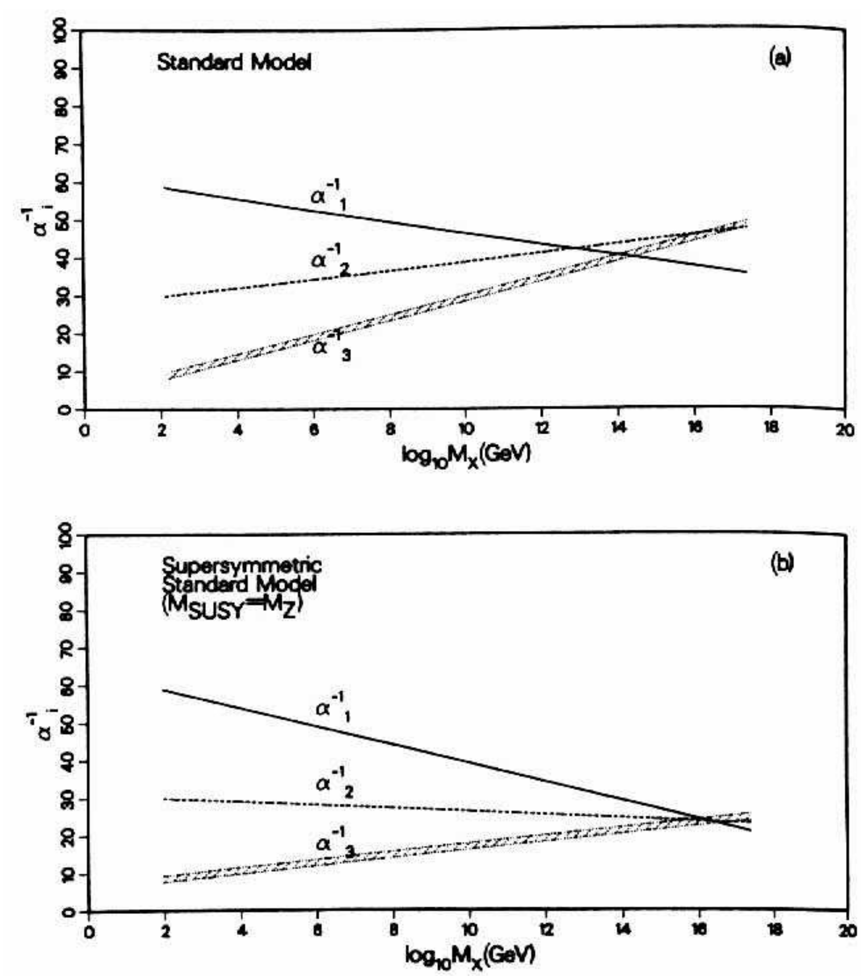
[For "cultural" observations, see Taubes, 1997.]

## Supersymmetry

Can also avoid Landau pole problem via grand unification and strings.

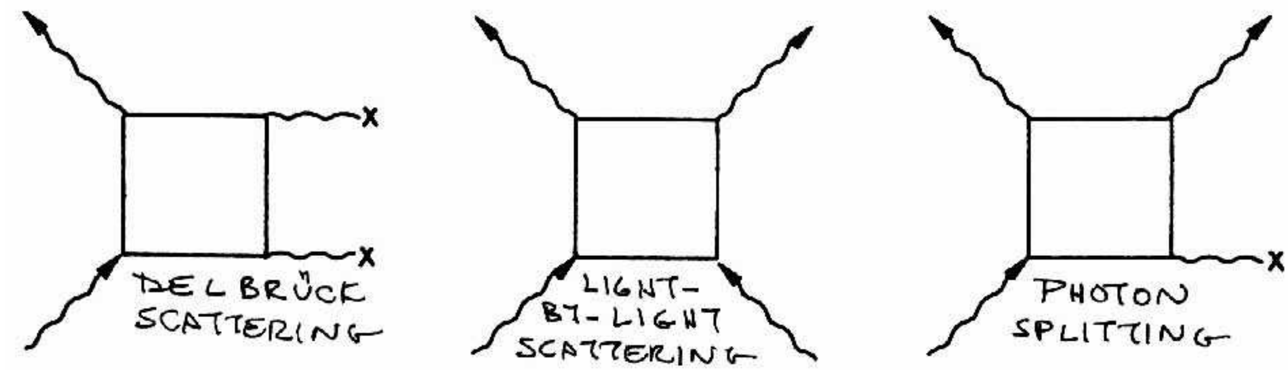
Elegant variant of grand unification invokes supersymmetry to bring the running of $\alpha_{\mathrm{QED}},\ \alpha_{\mathrm{strong}}$ and $\alpha_{\mathrm{weak}}$ together at a common energy.

[Dimopoulos, Raby & Wilczek, 1981]



## Boxes

Electromagnetic boxes observed via Delbrück scattering.




[Jarlskog *et al.*, 1983]

Light-by-light scattering with real photons not yet observed.

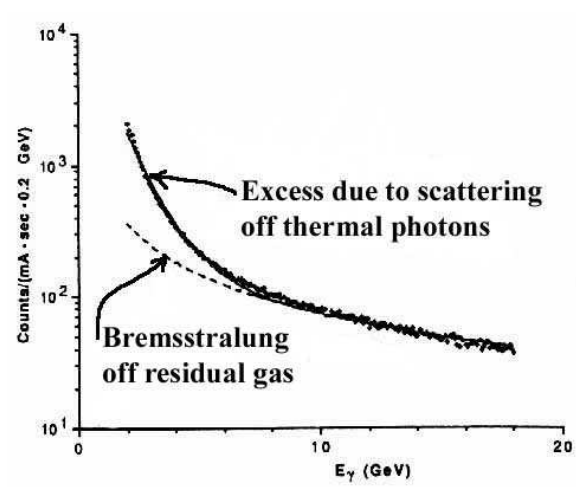
## Finite Temperature QED

Light-by-light scattering shifts Planck spectrum:

$$\frac{\Delta\lambda}{\lambda} \propto \alpha^2 \left(\frac{kT}{mc^2}\right)^4 \approx 10^{-35} \left(\frac{T}{300\text{K}}\right)^4, \text{ [Barton, 1990; Ravndal, 1997]}$$

Compton scattering of LEP beam off thermal photons:

[Dehning *et al.*, 1990]



# The Gauge Theory of Arbitrage

## Physics of Finance

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**We give a brief introduction to the Gauge Theory of Arbitrage [1]. Treating a calculation of net present values (NPV) and currencies exchanges as a parallel transport in some fibre bundle, we give geometrical interpretation of the interest rate, exchange rates and prices of securities as a proper connection components. This allows us to map the theory of capital market onto the theory of quantized gauge field interacted with a money flow field. The gauge transformations of the matter field correspond to a dilatation (redefinition) of security units which effect is eliminated by a proper tune of the connection. The curvature tensor for the connection consists of the excess returns to the risk-free interest rate for the local arbitrage operation. Free quantum gauge theory is equivalent to the assumption about the log-normal walks of assets prices. In general case the consideration maps the capital market onto QED, i.e. quantum system of particles with positive (securities) and negative ("debts") charges which interact with each other through electromagnetic field (gauge field of the arbitrage). In the case of a local virtual arbitrage opportunity money flows in the region of configuration space (money poor in the profitable security) while "debts" try to escape from the region. Entering positive charges and leaving negative ones screen up the profitable fluctuation and restore the equilibrium in the region where there is no arbitrage opportunity any more, i.e. speculators washed out the arbitrage opportunity.**

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## Sonoluminescence

In 1850, the Navier-Stokes equation was the "theory of everything", but it doesn't predict sonoluminescence. [Erber]

[Sonoluminescence is what makes nitroglycerine explode.]

- Preparata (1998): QED theory of water vapor predicts emission of light when water vapor condenses at density near 1 g/cm$^3$.
- Schwinger (1992): a bubble is an electromagnetic cavity; an imploding bubble will radiate away the changing, trapped zero-point energy.
- Liberati (1998): Imploding bubble $\Rightarrow$ rapidly changing index $\Rightarrow$ associated radiation.

  This relates to an earlier idea:
- Yablonovitch (1989): An accelerating boundary across which the index of refraction changes is a possible realization of the Hawking-Unruh effect, leading to conversion of QED vacuum fluctuations into real photons.

# The Hawking-Unruh Effect

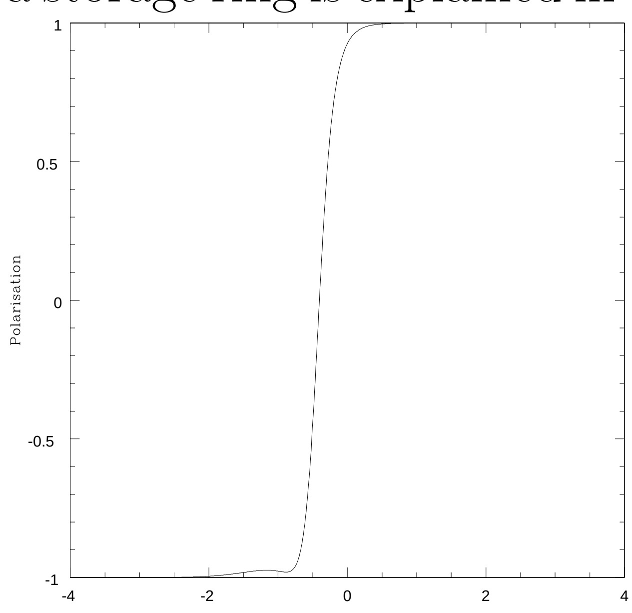
**Hawking** (1974): An observer outside a black hole experiences a bath of thermal radiation of temperature $T = \frac{\hbar g}{2\pi ck}$ ,

where $g$ is the local acceleration due to gravity.

**Unruh** (1976): According to the equivalence principle an accelerated observer in a gravity-free region should also experience a thermal bath with: $T = \frac{\hbar a}{2\pi ck}$ ,

where $a$ is the acceleration of the observer as measured in his instantaneous rest frame.

Bell (1983), Leinaas (1998), Unruh (1998): Incomplete polarization of electrons in a storage ring is explained in detail by Hawking-Unruh excitation.



# Strong-Field QED

For high acceleration, need strong electromagnetic field.

Strongest macroscopic electromagnetic fields are in lasers.

Tabletop teraWatt lasers can be focused to $> 10^{19}$ W/cm$^2$.

$\Rightarrow$ Electric fields $> 100$ GeV/cm.

[Photon number density $> 10^{27}$/cm$^3$.]

(Nonperturbative) physics described by two dimensionless measures of field strength:

$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{eE_{\rm rms}}{m\omega_0 c} = \frac{eE_{\rm rms}\lambda\!\!\!^-_0}{mc^2},$$

governs the importance of multiple photons in the initial state, and characterizes the "mass shift": $\overline{m} = m\sqrt{1+\eta^2}$. [Kibble, 1996]

$$\Upsilon = \frac{\sqrt{\langle (F^{\mu\nu} p_\nu)^2 \rangle}}{mc^2 E_{\rm crit}} = \frac{2p_0}{mc^2}\frac{E_{\rm rms}}{E_{\rm crit}} = \frac{2p_0}{mc^2}\frac{\lambda\!\!\!^-_C}{\lambda\!\!\!^-_0}\eta,$$

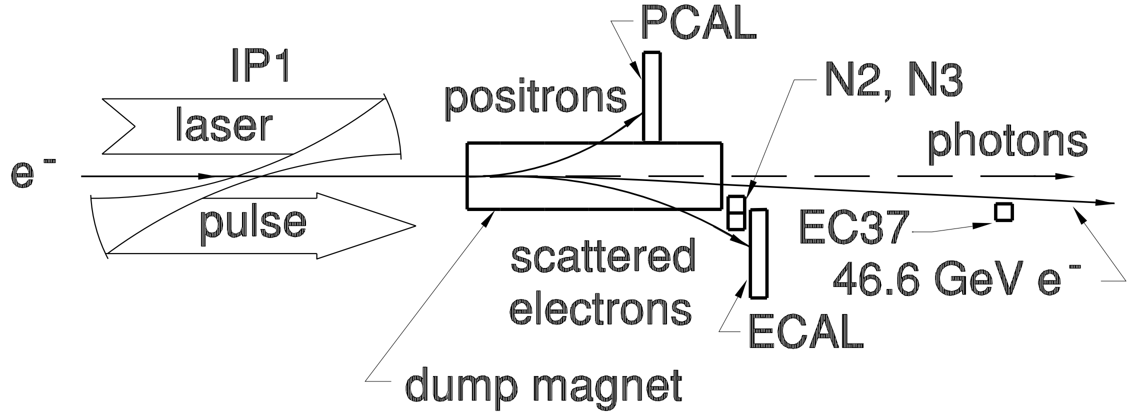
governs the importance of "spontaneous" pair creation, where $E_{\rm crit} = m^2c^3/e\hbar = mc^2/e\lambda\!\!\!^-_C = 1.3 \times 10^{16}$ V/cm.

# Where to Find Critical Fields

- The magnetic field at the surface of a neutron star approaches the critical field $B_{\mathrm{crit}} = 4.4 \times 10^{13}$ Gauss.
- During heavy-ion collisions where $Z_{\mathrm{total}} = 2Z > 1/\alpha$, the critical field can be exceeded and $e^+e^-$ production is expected.

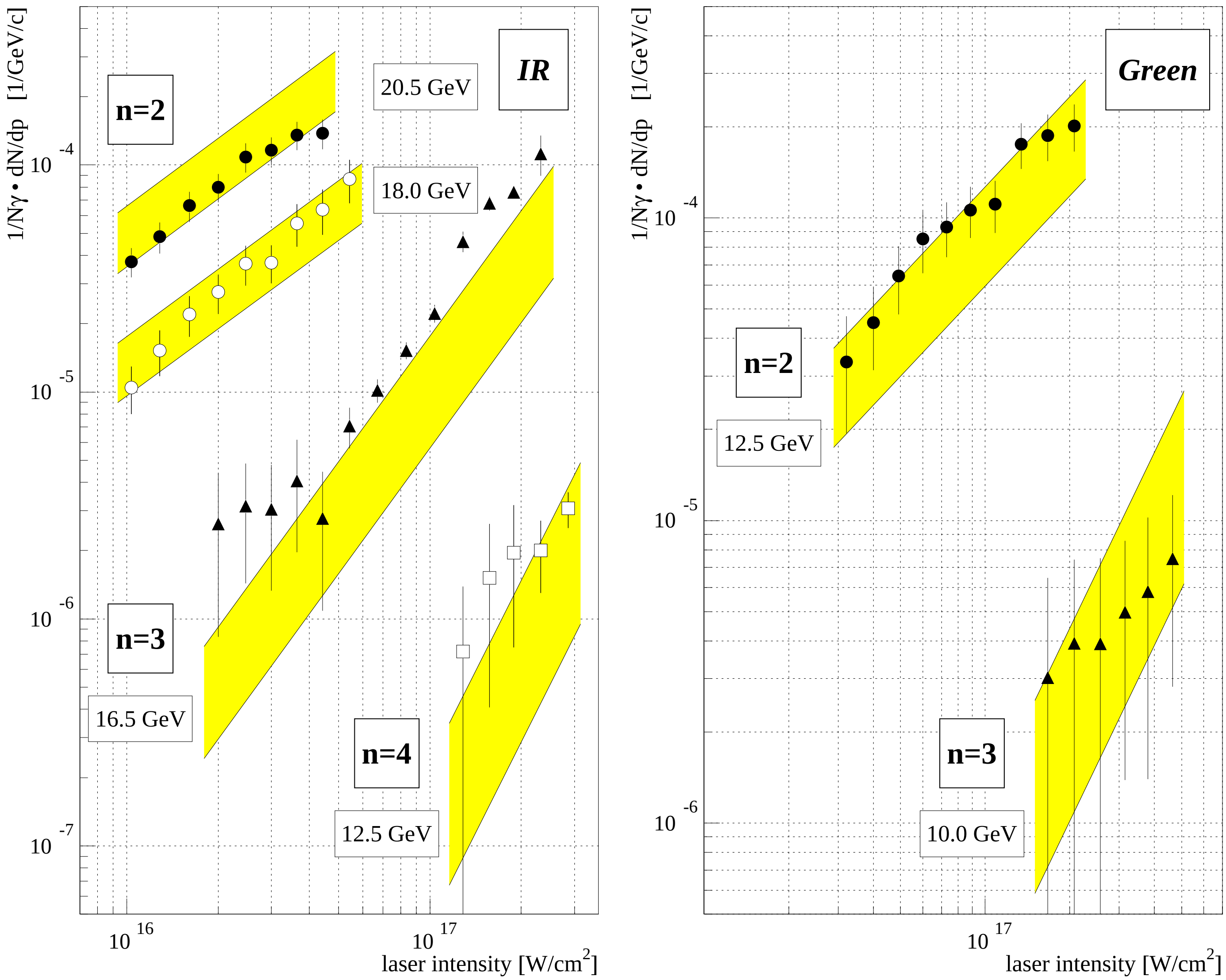
$$E_{\mathrm{max}} \approx \frac{2Ze}{\lambda_C^2} = 2Z\alpha E_{\mathrm{crit}}.$$

- Pomeranchuk (1939): The earth's magnetic field appears to be critical strength as seen by a cosmic-ray electron with $10^{19}$ eV.
- The electric field of a bunch at a future linear collider approaches the critical field in the frame of the oncoming bunch.
- The electric field of a focused teraWatt laser appears critical to a counterpropagating 50-GeV electron.



# Physics at High $\eta$: Nonlinear Compton Scattering

$$e + n\omega_0 \rightarrow e' + \omega \quad \text{[Bula \textit{et al.}, 1996]}$$




Normalized to total scattered photon rate

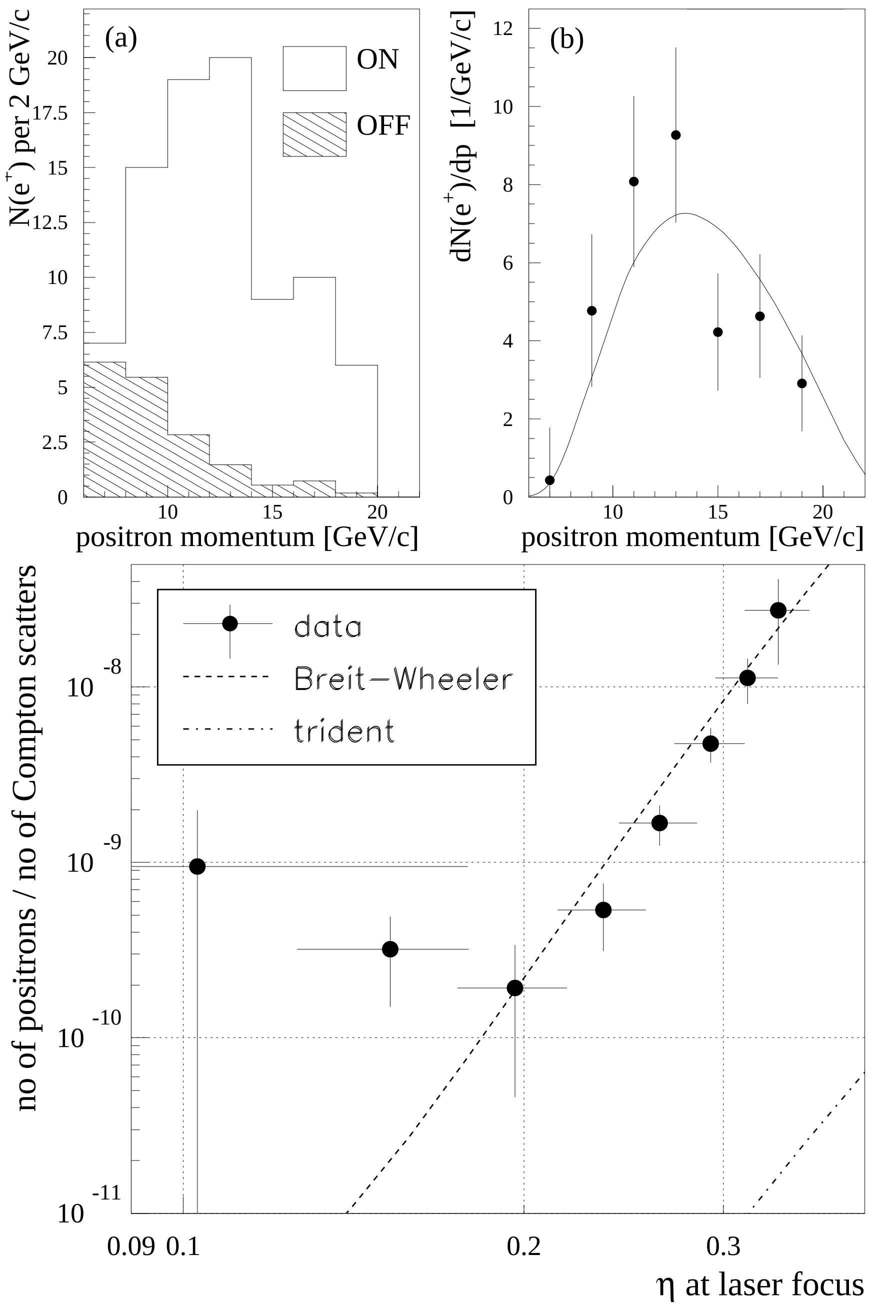
$\Rightarrow$ Rate(order $n$) $\propto I^{n-1}$.

Theory based on Volkov states of Dirac electron in a plane wave [Reiss (1962), Nikishov & Ritus (1964), Narozhny (1965)].

# Physics at High $\Upsilon$: Pair Creation by Light

Two step process: $e + \omega_0 \rightarrow e' + \omega$, then $\omega + n\omega_0 \rightarrow e^+e^-$.

$106 \pm 14$ signal positrons. [Burke *et al.*, 1997]




Rate $\propto \eta^{2n}$ where $n = 5.1 \pm 0.2$ (stat.) $^{+0.5}_{-0.8}$ (syst.)

$\Rightarrow$ 5 laser photons (process is below threshold for 1 photon).

# Strong Field Pair Creation as Barrier Penetration

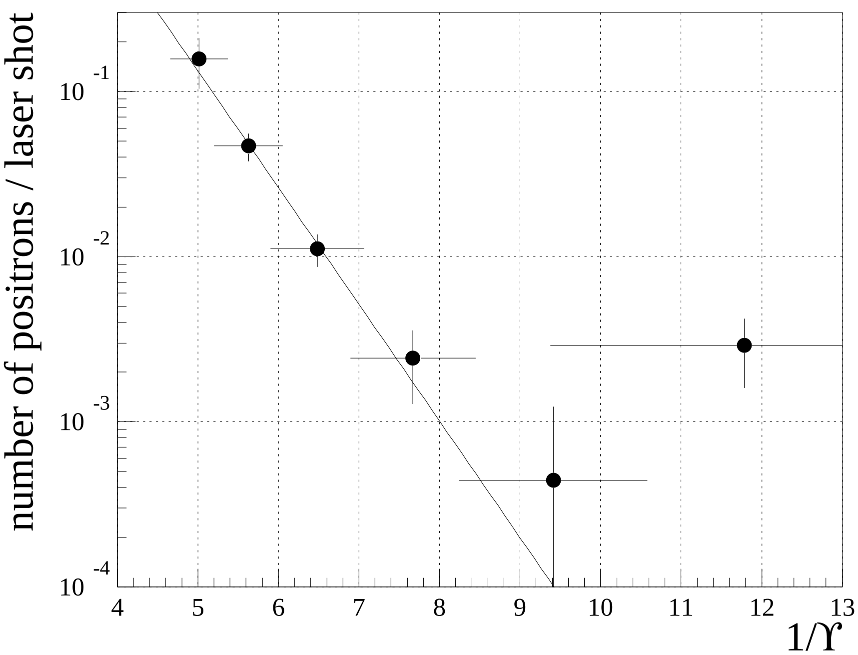
For a virtual $e^+e^-$ pair to materialize in a field $E$ the electron and positron must separate by distance $d$ sufficient to extract energy $2mc^2$ from the field:

$$eEd = 2mc^2.$$

The probability of a separation $d$ arising as a quantum fluctuation is related to penetration through a barrier of thickness $d$:

$$P \propto \exp\left(-\frac{d}{\lambda_C}\right) = \exp\left(-\frac{2m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{2E_{\rm crit}}{E}\right) = \exp\left(-\frac{2}{\Upsilon}\right).$$

[Sauter (1931), Heisenberg and Euler (1936), Schwinger (1951)]




$$R_{e^+} \propto \exp[(-1.8 \pm 0.2\,(\text{stat.}) \pm 0.2\,(\text{syst.}))/\Upsilon].$$

# Summary

- Higher-order QED (physics depending on high powers of $\alpha_{\mathrm{QED}}$) is very mature both experimentally and theoretically.

  New results will probe strong and electroweak corrections rather than yet higher orders of QED.

- Nonperturbative (strong-field) QED is still relatively young.

  New experiments involving intense laser beams at $\eta \approx 1$ and $\Upsilon \approx 1$ agree with existing theories.

  The frontier is at $\eta,\ \Upsilon \gg 1$.

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