Spin of the $\Lambda$ Hyperon via the Adair Method

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1 Problem

$\Lambda^0$ hyperons are produced by a pion beam in the reaction $\pi^- p \rightarrow K^0 \Lambda^0$, and observed by the decay $\Lambda^0 \rightarrow p\pi^-$ (which is a weak interaction that does not conserve parity). Let $J$ denote the spin of the $\Lambda$ (considered to be unknown in this problem, while the spins of the $\pi^-$, $p$ and $K^0$ are known), and $\theta$ be the angle of a decay product in the $\Lambda$ rest frame, relative to the direction of the $\Lambda$ in the lab frame. In the case where the $\Lambda$ is produced exactly along the beam direction, what are the possible values of $J_z$?

Show that for unpolarized beam protons, and for $\Lambda$’s produced along the beam direction, the $\Lambda$-decay angular distribution depends on $J$ according to

$$J = 1/2, \quad \text{isotropic},$$

$$J = 3/2, \quad 3\cos^2\theta + 1,$$

$$J = 5/2, \quad 5\cos^4\theta - 2\cos^2\theta + 1. \quad (1)$$


This problem is based on R.K. Adair, Angular Distribution of $\Lambda^0$ and $\theta^0$ Decays, Phys. Rev. 100, 1540 (1955), http://puhep.princeton.edu/~mcdonald/examples/EP/adair_pr_100_1540_55.pdf. The principle of this problem was used to determine that the $\Lambda^0$ has spin-1/2 by F. Eisler et al., Experimental Determinations of the $\Lambda^0$ and $\Sigma^-$ Spins, Nuovo Cim. 7 222 (1958), http://puhep.princeton.edu/~mcdonald/examples/EP/eisler_nc_7_222_58.pdf.
2 Solution

A two-particle state can only have orbital-angular-momentum component $L_z = 0$ along a $z$-axis.

If the $\Lambda^0$ moves along the beam axis, taken to be the $z$-axis, then so also does the $K^0$, and no matter what is their orbital angular momentum $L$, $L_z = 0$. Of course, the initial $\pi^- p$ state has $L_z = 0$, and $J_z = \pm 1/2$, since the pion is spinless and the proton has spin-1/2. Conservation of angular momentum then implies that $J_z = \pm 1/2$ for the final state; these two states are distinguishable, so it suffices to consider only one, say $J_z = 1/2$.

Similarly, since the initial state can only have $J = n/2$ for odd $n$ this also holds for the final state, which in turn implies that the spin of the $\Lambda^0$ is $m/2$ for odd $m$, since the $K^0$ is spinless.

1. $J_\Lambda = 1/2$.

In general, the decay final state $\pi^- p$ could have $L = 0$ or 1 such that $J = 1/2$. If the $\Lambda$ has $J_z = \pm 1/2$ in its rest frame, then this couples to the $L = 0$ $\pi^- p$ state according to

$$|1/2, 1/2\rangle = |0, 0\rangle|1/2, \pm 1/2\rangle,$$

(2)

and couples to the $\pi^- p$ states with orbital angular momentum $L = 1$ and (proton) spin $S = \pm 1/2$ according to

$$|1/2, 1/2\rangle = \sqrt{2/3}|1, 1\rangle|1/2, -1/2\rangle - \sqrt{1/3}|1, 0\rangle|1/2, 1/2\rangle,$$

(3)

$$|1/2, -1/2\rangle = -\sqrt{2/3}|1, -1\rangle|1/2, 1/2\rangle + \sqrt{1/3}|1, 0\rangle|1/2, -1/2\rangle,$$

(4)


The initial $J_z = \pm 1/2$ states, and the decay final states are all distinguishable by the proton spin component, so we have four amplitudes to consider,

$$\alpha|0, 0\rangle|1/2, 1/2\rangle - \beta\sqrt{1/3}|1, 0\rangle|1/2, 1/2\rangle,$$  

(5)

$$\beta\sqrt{2/3}|1, 1\rangle|1/2, -1/2\rangle,$$  

(6)

$$\alpha|0, 0\rangle|1/2, -1/2\rangle + \beta\sqrt{1/3}|1, 0\rangle|1/2, -1/2\rangle,$$  

(7)

$$-\beta\sqrt{2/3}|1, -1\rangle|1/2, 1/2\rangle,$$  

(8)

where $\alpha$ is the strength of the interaction with the $L = 0$ state, and $\beta$ is the strength of the interaction with the $L = 1$ state. We square amplitudes (5)-(8) and add to
find the angular distribution, noting that the orbital angular momentum states $|L, L_z\rangle$ correspond to spherical harmonics $Y^L_{L_z}(\theta, \phi)$, where $\theta$ is the angle of, say, the decay pion with respect to the $z$-axis in the $\Lambda$ rest frame.

$$Y^0_0 = \sqrt{\frac{1}{4\pi}}, \quad Y^\pm_1 = \mp \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi}, \quad Y^0_1 = \sqrt{\frac{3}{4\pi}} \cos \theta. \quad (9)$$

The four amplitudes (5)-(8) are then (after multiplying by $\sqrt{4\pi}$),

$$\alpha - \beta \sqrt{\frac{1}{3}} \cos \theta, \quad -\beta \sqrt{\frac{1}{3}} \sin \theta e^{i\phi}, \quad \alpha + \beta \sqrt{\frac{1}{3}} \cos \theta, \quad -\beta \sqrt{\frac{1}{3}} \sin \theta e^{-i\phi}. \quad (10)$$

Squaring, and adding, leads to the angular distribution

$$2|\alpha|^2 + 2 \frac{|\beta|^2}{3} (\sin^2 \theta + \cos^2 \theta) = 2|\alpha|^2 + 2 \frac{|\beta|^2}{3} = \text{isotropic}. \quad (11)$$

We note that the target protons needed to be unpolarized so that the cases of $J_z = \pm 1/2$ for the initial state are equally likely, and the cross terms between different $L$ in the final $\pi^- p$ state cancel out. We assume this holds for the cases of higher possible $\Lambda$ spin, and consider than contributions to the angular distribution from different $L$ separately.


In this case the orbital angular momentum of the $\pi^- p$ final state can be $L = 1$ or 2 such that $J = 3/2$. If the $\Lambda$ has $J_z = 1/2$ in its rest frame, then this couples to the $\pi^- p$ final states with orbital angular momentum $L = 1$ and (proton) spin $S = 1/2$ according to

$$|3/2, 1/2\rangle = \sqrt{\frac{1}{3}}|1, 1\rangle|1/2, -1/2\rangle + \sqrt{\frac{2}{3}}|1, 0\rangle|1/2, 1/2\rangle, \quad (12)$$

which implies an angular distribution proportional to

$$|Y^1_1|^2 + 2 |Y^0_1|^2 \propto \frac{\sin^2 \theta}{2} + 2 \cos^2 \theta \propto 3 \cos^2 \theta + 1. \quad (13)$$

Similarly, the coupling to the $\pi^- p$ final states with orbital angular momentum $L = 2$ is

$$|3/2, 1/2\rangle = \sqrt{\frac{3}{5}}|2, 1\rangle|1/2, -1/2\rangle - \sqrt{\frac{2}{5}}|2, 0\rangle|1/2, 1/2\rangle, \quad (14)$$

which implies an angular distribution of

$$3 |Y^2_2|^2 + 2 |Y^0_2|^2 \propto 3 \frac{15}{2} \sin^2 \theta \cos^2 \theta + 2 \frac{5}{4} (3 \cos^2 \theta - 1)^2 \propto 3 \cos^2 \theta + 1, \quad (15)$$

noting that

$$Y^1_2 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}, \quad Y^0_2 = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1). \quad (16)$$

Thus, either value of $L$ for the $\pi^- p$ final states leads to the same angular distribution, namely $3 \cos^2 \theta + 1$. 

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3. $J_A = 5/2$.

In this case the possible orbital angular momenta of the final $\pi^- p$ states are $L = 2$ and $3$.

We content ourselves with calculating only $L = 2$.

$$|5/2, 1/2\rangle = \sqrt{\frac{2}{5}}|2, 1\rangle|1/2, -1/2\rangle + \sqrt{\frac{3}{5}}|2, 0\rangle|1/2, 1/2\rangle,$$

which implies an angular distribution of

$$2 |Y_2^1|^2 + 3 |Y_2^0|^2 \propto 2 \frac{15}{2} \sin^2 \theta \cos^2 \theta + 3 \frac{5}{4} (3 \cos^2 \theta - 1)^2 \propto 5 \cos^4 \theta - 2 \cos^2 \theta + 1.$$