IL NUOVO CIMENTO VOL. XVI, N. 4 16 Maggio 1960

The Axial Vector Current in Beta Decay (*).

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(ricevuto il 19 Febbraio 1960)

Summary. — In order to derive in a convincing manner the formula of Goldberger and Treiman for the rate of charged pion decay, we consider the possibility that the divergence of the axial vector current in β-decay may be proportional to the pion field. Three models of the pion-nucleon interaction (and the weak current) are presented that have the required property. The first, using gradient coupling, has the advantage that it is easily generalized to strange particles, but the disadvantages of being unrenormalizable and of bringing in the vector and axial vector currents in an unsymmetrical way. The second model, using a strong interaction proposed by SCHWINGER and a weak current proposed by POLKINGHORNE, is renormalizable and symmetrical between V and A, but it involves postulating a new particle and is hard to extend to strange particles. The third model resembles the second one except that it is not necessary to introduce a new particle. (Renormalizability in the usual sense is then lost, however). Further research along these lines is suggested, including consideration of the possibility that the pion decay rate may be plausibly obtained under less stringent conditions.

(*) Supported in part by the Alfred P. Sloan Foundation and by the United States Air Force through the European Office, Air Research and Development Command.

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1. – Introduction.

The decay of the muon is the only process known experimentally in which the weak interactions can apparently be studied without complications due to the strong interactions. The electromagnetic corrections, moreover, are finite and have been calculated (1). All evidence so far supports the correctness of the following Lagrangian for the interaction:

$$\mathscr{L}(\mu \text{ decay}) = 2^{-\frac{1}{2}} G_\mu [\bar{\nu}\gamma_\alpha(1+\gamma_5)e][\bar{\nu}'\gamma_\alpha(1+\gamma_5)\mu]^+ + \text{Herm. conj.} \tag{1}$$

(The two neutrinos involved have been denoted by different symbols ν and ν' because we are not certain that they are identical, although they are both massless and they exhibit the same helicity). The value of the constant G_μ can be determined from the rate of decay of the muon according to the formula:

$$\Gamma_\mu = (192\pi^3)^{-1} G_\mu^2 m_\mu^5 (0.995\,6)\,, \tag{2}$$

which is well known, perhaps with the exception of the final factor, which gives the electromagnetic correction computed in Ref. (1). If we take (2) the muon lifetime Γ_μ^{-1} to be $2.208 \pm 0.003 \cdot 10^{-6}$ s and the mass m_μ to be 106.65 ± 0.01 MeV, then we get for the dimensionless quantity $G_\mu m_p^2$ the value:

$$G_\mu m_p^2 = 1.204 \pm 0.001 \cdot 10^{-5}\,,$$

where m_p is the proton mass and our units are such that $\hbar = c = 1$.

Now let us turn to those leptonic weak processes in which baryons and mesons are involved (bringing in the strong interactions), but in which there is no change of strangeness ($\Delta S = 0$). Experimentally, we deal with nuclear β decay (including K capture and inverse β decay), muon capture by nuclei, and the decay of the charged pion. It appears that all these processes can be described by an interaction Lagrangian of the form:

$$\mathscr{L}_{\text{int.}} = 2^{-\frac{1}{2}} G[V_\alpha + P_\alpha][\bar{p}\gamma_\alpha(1+\gamma_5)e + \bar{\nu}'\gamma_\alpha(1+\gamma_5)\mu]^+ + \text{Herm. conj.}\,, \tag{3}$$

where V_α and P_α are vector and pseudovector currents which can transform neutron into proton. As has, of course, been remarked (3,4), the experiments

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(1) S. M. BERMAN: *Phys. Rev.*, **112**, 267 (1958).
(2) Private communication from V. L. TELEGDI on the work of the Chicago group.
(3) R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958).
(4) E. C. G. SUDARSHAN and R. E. MARSHAK: *Phys. Rev.*, **109**, 1860 (1958).

suggest that $G(V_\alpha + P_\alpha)$ is very much like $G_\mu \bar{p}\gamma_\alpha(1+\gamma_5)n$, so that the weak interactions are essentially «universal» in their strength and form for the pairs νe, ν′μ, and pn. However, $G(V_\alpha + P_\alpha)$ need not be precisely of the form indicated; there may, for example, be other terms. It is with just this question of the structure of the currents, especially of P_α, that we are concerned here.

For the sake of definiteness, let us agree that the coefficient of $\bar{p}\gamma_\alpha n$ in V_α is unity—that is our definition of G. We also assume that the coefficient of $\bar{p}\gamma_\alpha\gamma_5 n$ in P_α is unity.

Because of the presence of strong interactions, we do not necessarily observe G directly. In the β decay of the neutron, for example, we can measure the matrix elements of GV_α and GP_α between free nucleon states with very little momentum transfer ($\leqslant 1$ MeV). In this limit we have:

$$\langle p|V_\alpha|n\rangle \to G_V \bar{u}_f \tau_+ \gamma_\alpha u_i , \tag{4a}$$

$$\langle p|P_\alpha|n\rangle \to -G_A \bar{u}_f \tau_+ \gamma_\alpha\gamma_5 u_i , \tag{4b}$$

where G_V and G_A are the conventional Fermi and Gamow-Teller coupling constants of nuclear β decay and u_i and u_f are the initial and final free nucleon spinors.

It is well known that the experimental value of G_V is remarkably close to that of G_μ and an explanation [3] of this fact has been offered based on two theoretical hypotheses:

a) Exact «universality» of strength: $G = G_\mu$.

b) The conserved vector current theory [3,5] of V_α, which gives $G_V = G$ (apart from electromagnetic corrections) as a consequence of the vanishing of the divergence $\partial_\alpha V_\alpha$.

So far the best evidence for this point of view is the *ft* value of the decay of ^{14}O, which is predicted to be $3\,001 \pm 6$ on the basis of *a*) and *b*) and the value of G_μ quoted above, while the experimental result [6] is $3\,088 \pm 56$. The theoretical prediction is subject to error only from the experimental muon lifetime and mass and from electromagnetic corrections to the decay of ^{14}O.

There is, of course, some discrepancy between theory and experiment, which is made worse if we accept Berman's estimate [1] of the electromagnetic corrections, which reduces the predicted *ft* value to 2917. If we take seriously

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[5] S. S. GERSHTEIN and J. B. ZELDOVICH: *Žurn. Eksp. Teor. Fiz.*, **29**, 698 (1955) (translation: *Sov. Phys. Journ. Exp. Theor. Phys.*, **2**, 576 (1957)).

[6] D. A. BROMLEY, E. ALMQUIST, H. E. GOVE, A. E. LITHERLAND, E. B. PAUL and A. J. FERGUSON: *Phys. Rev.*, **105**, 957 (1957).

this number, the experimental value, and the conserved vector current hypothesis, we obtain $G/G_\mu = 0.97 \pm 0.01$ rather than unity (*).

Other tests of the theory have been proposed ([3,7]) but not yet carried out. For the time being, let us suppose it to be correct and go on to inquire about the form of the pseudovector current P_α.

At one time it was suggested ([3]) that here too the renormalization factor might be unity. Some effort was put into a search for theories in which that would be true. Certain authors tried to find theories in which P_α would be divergenceless, by analogy with the vector case. The following points are now clear in connection with this type of investigation:

A) Experimentally ([8]) the quantity $-G_A/G_V$ is 1.25 ± 0.06, so that the axial vector renormalization factor is not unity, although it is not very far away.

B) The divergence $\partial_\alpha P_\alpha$ of the axial vector current cannot in any case be zero, because that would make the rate of decay of the charged pion vanish ([9]).

C) If, in some particular theoretical model, there is a limit in which $\partial_\alpha P_\alpha$ is zero, it is a delicate limit in which, for example, the nucleon mass or the pion mass vanishes; and the question of whether in this limit $-G_A/G$ is really unity must be carefully investigated for each model ([10]).

D) No one has found a theory in which a reasonable calculation of $-G_A/G$ can be made with present methods.

(*) *Note added in proof.* – Should this discrepancy be real, it would probably indicate a total or partial failure of the conserved vector current idea. It might also mean, however, that the current is conserved but with $G/G_\mu < 1$. Such a situation is consistent with universality if we consider the vector current for $\Delta S = 0$ and $\Delta S = 1$ together to be something like:

$$GV_\alpha + GV_\alpha^{(\Delta S=1)} = G_\mu \bar{p}\gamma_\alpha (n + \varepsilon \Lambda)(1 + \varepsilon^2)^{-\frac{1}{2}} + \dots ,$$

and likewise for the axial vector current. If $(1+\varepsilon^2)^{-\frac{1}{2}} = 0.97$, then $\varepsilon^2 = .06$, which is of the right order of magnitude for explaining the low rate of β decay of the Λ particle. There is, of course, a renormalization factor for that decay, so we cannot be sure that the low rate really fits in with such a picture.

([7]) M. GELL-MANN: *Phys. Rev.*, **111**, 362 (1958).

([8]) M. T. BURGY, V. E. KROHN, T. B. NOVEY, G. R. RINGO and V. L. TELEGDI: *Phys. Rev.*, **110**, 1214 (1958). See also C. S. WU: *Rev. Mod. Phys.*, **31**, 783 (1959).

([9]) J. C. TAYLOR: *Phys. Rev.*, **110**, 1216 (1958); M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev.*, **110**, 1478 (1958).

([10]) R. J. BLIN-STOYLE: *Nuovo Cimento*, **10**, 132 (1958); S. OKUBO: *Nuovo Cimento*, **13**, 292 (1959).

Despite the lack of success of the program just discussed, it has turned up at least three models in which $\partial_\alpha P_\alpha$, instead of vanishing, is proportional to a component of the pion field. This relation is interesting, not because it explains why $-G_A/G$ is fairly close to one, but because it gives a relation between the value of $-G_A$ and the rate of decay of the charged pion.

The connection of the formula

$$\partial_\alpha P_\alpha = -\frac{ia}{\sqrt{2}}\pi^- \tag{5}$$

with the rate of pion decay was discovered originally (*) for a particular model, in which the pion-nucleon strong interaction has the gradient form.

Our work on this model is an extension of that of NORTON and WATSON [11] and J. C. TAYLOR [12].

The formula relating $-G_A$ to the charged pion decay amplitude is essentially the one proposed by GOLDBERGER and TREIMAN [13], which gives remarkable agreement with experiment. We shall derive, in any theory for which eq. (5) is valid, an exact formula for pion decay, to which the equation of Goldberger and Treiman is a very plausible approximation.

We shall then investigate three models of strong and weak couplings of nucleons and pions that yield eq. (5). All of these models present some difficulties, however. None is a really convincing theory. We must therefore come back to the question of whether eq. (5) is really necessary in order to derive the result of GOLDBERGER and TREIMAN in a convincing manner.

2. – The rate of charged pion decay.

Suppose we have a theory of the strong interactions and a definition of the axial vector current such that eq. (5) holds. Then the matrix element of P_α for negative pion decay may be written:

$$\langle 0 | P_\alpha(x) | \pi^- \rangle = -\frac{a}{\sqrt{2}} \frac{q_\alpha}{m_\pi^2} \langle 0 | \pi^-(x) | \pi^- \rangle , \tag{6}$$

where q_α is the four-momentum of the pion, since, on taking the divergence of both sides, we get back just eq. (5) between the pion state and the vacuum. Note $q^2 = -m_\pi^2$.

(*) By R. P. FEYNMAN, with some assistance from one of us (M. G.-M.).

(11) R. E. NORTON and W. K. R. WATSON: *Phys. Rev.*, **110**, 996 (1958).

(12) J. C. TAYLOR: quoted in ref. (17). See also R. F. STREATER and J. C. TAYLOR: *Nucl. Phys.*, **7**, 276 (1958).

(13) M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev.*, **110**, 1178 (1958). A recent criticism of their paper has been given by R. F. SAWYER: *Phys. Rev.*, **116**, 231 (1959); like us, SAWYER seeks a better derivation of their result.

The pion field operator $\pi^-(x)$ may be written as the product of a field renormalization factor, conventionally called $\sqrt{Z_3}$, and the renormalized operator $\pi_r^-(x)$, for which the matrix element between the pion state and the vacuum is just the same as that of a free field between a free particle state and the free vacuum. So we have for the matrix element of P_α in pion decay the following formula in terms of a $\sqrt{Z_3}$:

$$\langle 0 | P_\alpha(x) | \pi^- \rangle = -\frac{a\sqrt{Z_3}}{\sqrt{2}} \frac{q_\alpha}{m_\pi^2} \langle 0 | \pi_r^-(x) | \pi^- \rangle . \tag{7}$$

Now we may also evaluate $-G_A/G$ in terms of $a\sqrt{Z_3}$. We take the divergence of both sides of eq. (4*b*) in the limit of very small momentum transfer k (final momentum minus initial momentum) and we have:

$$G \langle p | \partial_\alpha P_\alpha | n \rangle \to -G_A(-ik_\alpha)\bar{u}_f \tau_+ \gamma_\alpha \gamma_5 u_i = 2m(-G_A)\bar{u}_f \tau_+ \gamma_5 u_i , \tag{8}$$

where m is the nucleon mass. If we are to apply eq. (5) we must calculate $\langle p | \pi^- | n \rangle$ in the limit of zero momentum transfer. Now in the neighborhood of $k^2 = -m_\pi^2$, we know this matrix element to be expressible in terms of the renormalized coupling constant g_1 as follows:

$$\langle p | \pi^- | n \rangle \approx \sqrt{Z_3}(k^2 + m_\pi^2)^{-1} i\sqrt{2}\, g_1 \bar{u}_f \tau_+ \gamma_5 u_i . \tag{9}$$

To make this formula correct at all values of k^2, we must simply replace the free propagator $(k^2 + m_\pi^2)^{-1}$ of the meson by the exact renormalized propagator, which we may call $(k^2 + m_\pi^2)^{-1} d_\pi(k^2)$, and the free vertex $\tau_+ \gamma_5$ by the exact renormalized vertex, which we may call $\tau_+ \gamma_5 F_\pi(k^2)$. The « form factors » $d_\pi(k^2)$ and $F_\pi(k^2)$ are both unity at $k^2 = -m_\pi^2$. We have, then, as $k^2 \to 0$, the result:

$$\langle p | \partial_\alpha P_\alpha | n \rangle \to \frac{ia}{\sqrt{2}} \frac{\sqrt{Z_3}}{m_\pi^2} d_\pi(0) F_\pi(0)\, i\sqrt{2}\, g_1 \bar{u}_f \tau_+ \gamma_5 u_i . \tag{10}$$

Comparing this equation with eq. (8), we find:

$$a\sqrt{Z_3} = -\frac{2m}{g_1} m_\pi^2 \left(-\frac{G_A}{G}\right) \frac{1}{d_\pi(0) F_\pi(0)} . \tag{11}$$

The unknown quantity in our formula (7) for the pion decay matrix element is now evaluated and we may calculate the rate of the process $\pi^- \to \mu^- + \bar{\nu}$,

which gives essentially the reciprocal lifetime of the charged pion:

$$\Gamma_\pi = \left(16\pi^2 \frac{g_1^2}{4\pi}\right)^{-1} (G_A m^2)^2 m_\pi \frac{m_\mu^2}{m^2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 [d_\pi(0) F_\pi(0)]^{-2} . \tag{12}$$

Except for the final factor, this is the same as the formula given by GOLDBERGER and TREIMAN ([13]). Their derivation, based on the conventional pseudoscalar theory of the strong interactions and the conventional definition $\bar{p}\gamma_\alpha\gamma_5 n$ of the current P_α, involves several violent approximations which are not really justified. The formula, however, is in excellent agreement with experiment. The measurements ([11]) give:

$$\frac{\Gamma_\pi}{m_\pi} = (1.84 \pm 0.04)\cdot 10^{-16}, \tag{13}$$

while eq. (12) yields:

$$\frac{\Gamma_\pi}{m_\pi} = [d_\pi(0) F_\pi(0)]^{-2} \{(1.56 \pm 0.2)\cdot 10^{-16}\}, \tag{14}$$

with $g_1^2/4\pi = 15 \pm 2$.

In the work of GOLDBERGER and TREIMAN, it was very mysterious that the agreement of the figures should be so close. If, however, eq. (12) is derived, as above, from a theory in which eq. (5) holds, then the discrepancy is to be attributed solely to the form factor $[d_\pi(0) F_\pi(0)]^{-2}$, which we know equals unity when the argument is $-m_\pi^2$. We would not be surprised if at zero the departure from unity amounts to only twenty percent or so.

We must be careful not to exaggerate the advantage of models in which eq. (5) holds. It can be shown (*) that in *any* theory that predicts a non-vanishing rate of pion decay we can obtain an *exact* equation analogous to (12), with the form factor $d_\pi(0)F_\pi(0)$ replaced by a general «form factor» $\varphi(0)$, where φ, like $d_\pi F_\pi$, equals unity at the value $-m_\pi^2$ of its argument. The difference between one theory and another lies merely in the question of whether this general «form factor» φ is likely to be slowly varying. If the theory is such that eq. (5) is valid, so that $\varphi = d_\pi F_\pi$, then it is not unreasonable that φ be slowly varying. In the conventional theory, where φ is something much more complicated, the conclusion is much less plausible. In any case, we cannot exclude the possibility that the formula of Goldberger and Treiman

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([14]) As listed by K. M. CROWE: *Nuovo Cimento*, **5**, 541 (1957).

(*) See forthcoming article by BERNSTEIN, FUBINI, GELL-MANN and THIRRING.

is approximately valid even in the conventional theory, for which they tried to derive it.

In the subsequent sections, we shall study models in which eq. (5) actually holds, but we must bear in mind the question of whether such an assumption is really necessary in order to explain what seems to be an experimental fact, that $\varphi(0)$ is close to unity.

In order to study the models conveniently, and in order to show the relation between the vector and the pseudovector currents, let us now review some formalism.

3. – The divergence of a current and gauge transformations [15].

Let us consider a theory of the strong interactions derivable from a Lagrangian density $\mathscr{L}$ expressed in terms of field components ψ_i and their first spatial derivatives $\partial_\alpha \psi_i$. Then the equations of motion of Lagrange are (*):

$$\frac{\delta \mathscr{L}}{\delta \psi_i} = \partial_\alpha \frac{\delta \mathscr{L}}{\delta(\partial_\alpha \psi_i)} . \tag{15}$$

Suppose we now subject each field component $\psi_i(x)$ to an infinitesimal local gauge transformation:

$$\psi_i(x) \to \psi_i(x) + \Lambda(x) F_i[\psi_1(x), \psi_2(x), \ldots] . \tag{16}$$

with a gauge function Λ. Then we may examine the variation of $\mathscr{L}$ under this change and we find, always to first order-:

$$\delta \mathscr{L} = \frac{\delta \mathscr{L}}{\delta \Lambda} \Lambda + \frac{\delta \mathscr{L}}{\delta \partial_\alpha \Lambda} \partial_\alpha \Lambda , \tag{17}$$

where

$$\frac{\delta \mathscr{L}}{\delta \Lambda} = \frac{\delta \mathscr{L}}{\delta \psi_i} F_i + \frac{\delta \mathscr{L}}{\delta \partial_\alpha \psi_i} \partial_\alpha F_i , \tag{18}$$

and

$$\frac{\delta \mathscr{L}}{\delta \partial_\alpha \Lambda} = \frac{\delta \mathscr{L}}{\delta \partial_\alpha \psi_i} F_i . \tag{19}$$

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[15] Our point of view in this section has much in common with that of S. GLASHOW: *Nucl. Phys.*, **10**, 107 (1959), as well as that of S. BLUDMAN: *Nuovo Cimento*, **9**, 433 (1958). See, also, the earlier work of J. SCHWINGER: ref. [16].

(*) Our discussion is classical. We ignore certain complications that may arise in quantum mechanics from the non-commutation of boson fields and their canonical momenta, making necessary a careful ordering of the operators.

Using (8), (11) and (12), we see immediately that we have Lagrange's equation for $\Lambda(x)$:

$$\frac{\delta \mathscr{L}}{\delta \Lambda} = \partial_\alpha \frac{\delta \mathscr{L}}{\delta \partial_\alpha \Lambda} . \tag{20}$$

Now suppose that under a particular transformation with infinitesimal gauge function Λ taken *independent of co-ordinates*, the Lagrangian $\mathscr{L}$ is *invariant*. Then $\delta \mathscr{L}/\delta \Lambda$ vanishes and thus:

$$\partial_\alpha \frac{\delta \mathscr{L}}{\delta \partial_\alpha \Lambda} = 0 . \tag{21}$$

We may then identify $\delta \mathscr{L}/\delta \partial_\alpha \Lambda$ as the current which is conserved as a result of the invariance of the theory under the gauge transformation with constant Λ.

Consider, for example, the conservation of baryons. Let each baryon field acquire a factor $(1 + i\Lambda_1(x))$, while the meson fields are left unchanged. The baryon current $\propto \delta \mathscr{L}/\delta \partial_\alpha \Lambda_1$ is then conserved.

For conservation of charge, we let each field acquire a factor $(1 + iQ\Lambda_2(x))$, where Q is the charge destroyed by the field. The Lagrangian is chosen invariant under this for constant Λ_2. The electric current $\propto \delta \mathscr{L}/\delta \partial_\alpha \Lambda_2$ is then conserved.

We note that we are working only with the Lagrangian of the strong interactions; that is to say, we are ignoring all higher effects of the electromagnetic field. We are therefore not concerned, at the moment, with the more general gauge transformation that includes photons and that also leaves the electric current exactly conserved.

For conservation of isotopic spin, we consider rotations in isotopic space with an isotopic vector gauge function $\boldsymbol{u}$. For example, for the nucleon field $N(x)$ and the pion field $\boldsymbol{\pi}(x)$, we have:

$$\begin{cases} N \to (1 + i\boldsymbol{\tau}\cdot\boldsymbol{u})N , \\ \boldsymbol{\pi} \to \boldsymbol{\pi} - 2\boldsymbol{u}\times\boldsymbol{\pi} . \end{cases} \tag{22}$$

With the Lagrangian invariant under these rotations for $\boldsymbol{u}$ constant, we have conservation of the isotopic spin current $\propto \delta \mathscr{L}/\delta \partial_\alpha \boldsymbol{u}$.

In the conserved vector current theory of the weak couplings, $V_\alpha(x)$ is simply the + component of an isotopic vector current $\boldsymbol{V}_\alpha(x)$ which is equal to the isotopic spin current:

$$\boldsymbol{V}(x) = i \frac{\delta \mathscr{L}}{\delta \partial_\alpha \boldsymbol{u}} = \bar{N}\boldsymbol{\tau}\gamma_\alpha N + 2i\boldsymbol{\pi}\times\partial_\alpha\boldsymbol{\pi} + \dots , \tag{23}$$

where the terms we have written explicitly come from the free Lagrangian of N and $\boldsymbol{\pi}$.

Now for the axial vector weak current $P_\alpha(x)$ it is reasonable to suppose that it too is the + component of an isotopic vector $\boldsymbol{P}_\alpha(x)$ which can be generated from $\mathscr{L}$ by a gauge transformation with a gauge function $\boldsymbol{v}(x)$ that is a pseudoscalar in space and an isotopic vector:

$$\boldsymbol{P}_\alpha(x) = i\frac{\delta\mathscr{L}}{\delta\partial_\alpha\boldsymbol{v}}. \tag{24}$$

We are not, however, free to suppose that this current is divergenceless, *i.e.* that the Lagrangian is invariant under our gauge transformation with constant $\boldsymbol{v}$. Thus in place of the conservation law (14) we must use the more general formula (13), which gives us for the divergence of the pseudoscalar current the result:

$$\partial_\alpha\boldsymbol{P}_\alpha = i\frac{\delta\mathscr{L}}{\delta\boldsymbol{v}}. \tag{25}$$

Let us take as an example the conventional pseudoscalar theory of nucleons and pions with Lagrangian density:

$$\mathscr{L}_{ps} = -\bar{N}(\gamma\partial + m_0 - ig_0\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)N - \frac{\mu_0^2\pi^2}{2} - \frac{(\partial\boldsymbol{\pi})^2}{2} - \lambda_0(\pi^2)^2. \tag{26}$$

If we wish to have for our axial vector current P_α just the simple form $\bar{N}\boldsymbol{\tau}\gamma_\alpha\gamma_5 N$, then we take for our gauge transformation the following:

$$N \to (1 + i\boldsymbol{\tau}\cdot\boldsymbol{v}\gamma_5)N,$$

$$\boldsymbol{\pi} \to \boldsymbol{\pi}.$$

We then obtain for $\partial_\alpha\boldsymbol{P}_\alpha$ the result:

$$\frac{i\delta\mathscr{L}_{ps}}{\delta\boldsymbol{v}} = 2m_0\bar{N}\boldsymbol{\tau}\gamma_5 N - 2ig_0\boldsymbol{\pi}\bar{N}N.$$

If the gauge transformation is chosen instead to be this one:

$$N \to (1 + i\boldsymbol{\tau}\cdot\boldsymbol{v}\gamma_5)N,$$

$$\boldsymbol{\pi} \to \boldsymbol{\pi} + \frac{2m_0}{g_0}\boldsymbol{v},$$

then P_α is $\bar{N}\boldsymbol{\tau}\gamma_\alpha\gamma_5 N - (2im_0/g_0)\,\partial_\alpha\boldsymbol{\pi}$ and its divergence is:

$$\frac{i\,\delta\mathscr{L}_{ps}}{\delta v} = -2ig_0\boldsymbol{\pi}\bar{N}N - \frac{2m_0}{g_0}\mu_0^2 i\boldsymbol{\pi} - \frac{8\lambda_0 m_0}{g_0} i\pi^2\boldsymbol{\pi}\,.$$

In neither of these two cases does the divergence seem to have any simple properties. If, however, we change the theory of the strong interactions, we may find a gauge transformation that yields an axial vector current with a simple divergence; in fact we may find a current for which

$$\partial_\alpha \boldsymbol{P}_\alpha = ia\boldsymbol{\pi}\,,$$

giving eq. (5) and implying eq. (12) that explains the pion lifetime.

Evidently what we require is a Lagrangian for which there is a gauge transformation with pseudoscalar gauge vector $\boldsymbol{v}$ such that:

$$\frac{\delta\mathscr{L}}{\delta\boldsymbol{v}} = a\boldsymbol{\pi}\,. \tag{28}$$

For constant $\boldsymbol{v}$, then, the Lagrangian must be nearly invariant, with the gauge transformation adding only the term $a\,\boldsymbol{\pi}\cdot\boldsymbol{v}$.

Let us now examine the models so far discovered in which that happens.

4. – The gradient coupling model.

The first model to be found makes use of the gradient coupling theory. If we treat just nucleons and pions for simplicity, the Lagrangian is:

$$\mathscr{L}_1 = -\bar{N}(\gamma\partial + m_0 + if_0\boldsymbol{\tau}\cdot\partial_\alpha\boldsymbol{\pi}\gamma_\alpha\gamma_5)N - \frac{(\partial\boldsymbol{\pi})^2}{2} - \frac{\mu_0^2\pi^2}{2}\,. \tag{29}$$

Except for the last term it is invariant under the gauge transformation:

$$\begin{cases} N \to N\,, \\ \boldsymbol{\pi} \to \boldsymbol{\pi} + f_0^{-1}\boldsymbol{v}\,, \end{cases} \tag{30}$$

when $\boldsymbol{v}$ is constant and the last term gives just $a\boldsymbol{\pi}\cdot\boldsymbol{v}$ with:

$$a = -\frac{\mu_0^2}{f_0}\,. \tag{31}$$

The current $\boldsymbol{P}_\alpha$ is, of course, given by:

$$\boldsymbol{P}_\alpha = i\frac{\delta \mathscr{L}_1}{\delta \partial_\alpha \boldsymbol{v}} = \bar{N}\boldsymbol{\tau}\gamma_\alpha\gamma_5 N - \frac{i}{f_0}\partial_\alpha \boldsymbol{\pi}\,. \tag{32}$$

Comparing eq. (31) with eq. (11) we find:

$$-\frac{G_A}{G} = \frac{\mu_0^2}{m_\pi^2}\frac{\sqrt{Z_3}f_1}{f_0}\,d_\pi(0)\,F_\pi(0)\,, \tag{33}$$

where $f_1 = g_1/2m$ is the renormalized version of f_0. This relation can, of course, be proved directly for the gradient coupling theory with the axial vector current given in (32). In fact, there are two independent relations in the theory:

$$-\frac{G_A}{G} = \frac{f_1}{f_0\sqrt{Z_3}}\,F_\pi(0)\,, \tag{34}$$

and

$$\frac{Z_3\mu_0^2}{m_\pi^2}\,d_\pi(0) = 1\,, \tag{35}$$

the product of which gives eq. (33). Both are easy to prove. The first follows from the similarity of the weak pseudovector current and the source of the pion field. (The term in $\partial_\alpha\boldsymbol{\pi}$ in the weak current contributes nothing at zero energy.) The second relation obtains because at zero momentum the correction to the free meson propagator vanishes, since the source of the field is the divergence of $\bar{N}\boldsymbol{\tau}\gamma_\alpha\gamma_5 N$.

The gradient coupling model has two weak points. First, as is well known, there are violent divergences in every term of the perturbation expansion. If these were to be expressed as renormalizations, it would require the renormalization of an infinite number of parameters. Of course, we could simply introduce a cut-off, but then all quantities of physical interest would depend strongly on the cut-off (at least in the perturbation expansion) and the formal manipulations of the theory, such as we have carried out above, are not obviously meaningful.

The second point, which might not be serious, is that in our introduction of the weak currents by the gauge transformations (22) and (30), there is no similarity whatever between the gauges that generate the vector and the pseudovector currents. In the transformations (30) the term $\gamma_\alpha\gamma_5$ in the weak current for the nucleons is generated from the *coupling term* of the Lagrangian (29), while the corresponding vector term γ_α is, of course, generated from the free Lagrangian.

Yet we have evidence that the weak interactions are symmetrical between V and A, particularly their apparent equality of strength and the fact that for the leptons, which have no strong couplings, the weak coupling is just $\gamma_\alpha(1+\gamma_5)$.

5. – The σ model.

We have another example of a theory in which eq. (5) holds, if we take a Lagrangian for the strong interactions that is essentially one proposed by SCHWINGER ([16]) and then for the axial vector current the form suggested by POLKINGHORNE ([17]).

Again, for simplicity, we restrict ourselves to nucleons and pions only, except that we introduce (following SCHWINGER) a new scalar meson σ, with isotopic spin zero. It has strong interactions, and thus might easily have escaped observation if it is much heavier than π, so that it would disintegrate immediately into two pions. It would appear experimentally as a resonant state of two pions with $J=0$, $I=0$.

We take for our Lagrangian the following one, which leads to a renormalizable theory of the strong interactions:

$$\mathscr{L}_2 = -\bar{N}[\gamma\hat{\partial} + m_0 - g_0(\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)]N - \frac{(\hat{\partial}\boldsymbol{\pi})^2}{2} - \frac{(\hat{\partial}\sigma)^2}{2} - \frac{\mu_0^2\pi^2}{2} - \left(\mu_0^2 + \frac{2\lambda_0}{f_0^2}\right)\frac{\sigma^2}{2} - \lambda_0\left[(\pi^2+\sigma^2)^2 - \frac{2}{f_0}\sigma(\sigma^2+\pi^2)\right], \tag{36}$$

where $f_0 = g_0/2m_0$.

We have the usual pseudoscalar theory of the pion, with the σ added in a rather symmetrical way. The nature of the symmetry is made much clearer if we perform a translation of the field variable σ and re-express the Lagrangian in terms of the variable:

$$\sigma' \equiv \sigma - \frac{1}{2f_0}. \tag{37}$$

We have:

$$\mathscr{L}_2 = -\bar{N}[\gamma\hat{\partial} - g_0(\sigma' + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)]N - \frac{(\hat{\partial}\boldsymbol{\pi})^2}{2} - \frac{(\partial\sigma')^2}{2} - \frac{\mu_0^2}{2}(\pi^2+\sigma'^2) - \lambda_0\left[\pi^2+\sigma'^2 - \frac{1}{4f_0^2}\right]^2 - \frac{\mu_0^2}{2f_0}\sigma', \tag{38}$$

([16]) J. SCHWINGER: *Ann. Phys.*, **2**, 407 (1957).

([17]) J. C. POLKINGHORNE: *Nuovo Cimento*, **8**, 179, 781 (1958).

apart from an additive constant. It is evident at once that only the last term breaks the symmetry under the gauge transformation:

$$\text{(39)} \qquad \begin{cases} N \to (1 + i\boldsymbol{\tau}\cdot\boldsymbol{v}\gamma_5)N\,, \\ \boldsymbol{\pi} \to \boldsymbol{\pi} - 2\boldsymbol{v}\sigma'\,, \\ \sigma' \to \sigma' + 2\boldsymbol{v}\cdot\boldsymbol{\pi}\,, \end{cases}$$

with $\boldsymbol{v}$ constant. It is easy to see also that if the last term were absent the symmetry (39) would prevent the nucleon having any mass.

We now construct the pseudovector weak current from the same gauge transformation. We find:

$$\text{(40)} \qquad i\frac{\delta\mathscr{L}_2}{\delta\boldsymbol{v}} = -i\frac{\mu_0^2}{f_0}\boldsymbol{\pi},$$

so that eq. (27) holds with $a = -\mu_0^2/f_0$ as for the gradient coupling model. The current $\boldsymbol{P}_\alpha$ comes out:

$$\text{(41)} \qquad \boldsymbol{P}_\alpha = \bar{N}\boldsymbol{\tau}\gamma_\alpha\gamma_5 N + 2i(\sigma'\partial_\alpha\boldsymbol{\pi} - \boldsymbol{\pi}\partial_\alpha\sigma') = \\ = \bar{N}\boldsymbol{\tau}\gamma_\alpha\gamma_5 N + 2i(\sigma\partial_\alpha\boldsymbol{\pi} - \boldsymbol{\pi}\partial_\alpha\sigma) - \frac{i}{f_0}\partial_\alpha\boldsymbol{\pi}\,.$$

This time the gauge transformation that yields the axial vector current is closely related to the one (eq. (22)) that gives the vector current. Together, in fact, they form the generators of the rotation group in a four-dimensional Euclidean space. It is evident that, apart from the last term, the Lagrangian of eq. (38) is invariant under these four-dimensional rotations when the functions $\boldsymbol{u}$ and $\boldsymbol{v}$ are constant. The last term breaks the four-dimensional symmetry, but leaves the three-dimensional symmetry unchanged.

We may, if we like, consider a rotation in four dimensions that is a product of the rotations (22) and (39) with $\boldsymbol{u} = \boldsymbol{v} = \boldsymbol{w}$. We have:

$$\text{(42)} \qquad \begin{cases} N \to [1 + i\boldsymbol{\tau}\cdot\boldsymbol{w}(1 + \gamma_5)]N\,, \\ \boldsymbol{\pi} \to \boldsymbol{\pi} - 2\boldsymbol{w}\sigma' - 2\boldsymbol{w}\times\boldsymbol{\pi}\,, \\ \sigma' \to \sigma' + 2\boldsymbol{w}\cdot\boldsymbol{\pi}\,. \end{cases}$$

It is this rotation that generates the complete weak current $\boldsymbol{P}_\alpha + \boldsymbol{V}_\alpha$.

We see that if the mesons are taken out of the theory, then the transformation (42) works only on the free nucleon Lagrangian and we generate a weak current equal to $\bar{N}\tau_\alpha(1+\gamma_5)N$, which resembles the lepton weak current. Thus

the lack of symmetry between V and A mentioned in connection with the gradient coupling theory is not present here.

We don't have the divergence difficulty either—the present model is fully renormalizable. Moreover, the various matrix elements of the weak current seem to come out finite as well; even the renormalization factor $-G_A/G$ is finite (*).

Note that since $a = -\mu_0^2/f_0$ in both theories, eq. (33) is valid for both. (At first sight, it may look as if the individual theorems (34) and (35) also hold in the σ model, but in fact they don't work in perturbation theory.)

In view of eq. (33), which expresses $-G_A/G$ in terms of several divergent quantities, it may appear rather remarkable that it is finite. In particular, the reader may wonder what cancels the quadratic divergences of μ_0^2/m_π^2. The answer is that f_1/f_0 is the product of g_1/g_0 and m_0/m_1 and that in the σ model the quantity m_0/m_1 possesses quadratic divergences, even in second order. They come from « tadpole » diagrams in which a σ meson, emitted by the nucleon, turns into a nucleon and antinucleon that eat each other. It is the scalar, $I = 0$ quality of σ that makes such diagrams possible.

The σ model, although it has some agreable features that we have mentioned, is quite artificial. A new particle is postulated, for which there is no experimental evidence. It is true that if σ had a high enough mass it would not be easily detectable and that the theory allows for different π and σ masses, but we know of no theoretical reason for the mass of σ to come out very high.

The fact that the σ coupling is responsible for the nucleon mass is a curious property of the model. Unless we can explain all masses, or at least all baryon masses, in a similar way, it is not very satisfactory.

In any case, we are faced with the problem of extending our invariance under the $\boldsymbol{v}$ transformation to the strange particles. If we want to preserve the relation (27), we must add no new terms that violate the invariance for constant $\boldsymbol{v}$.

Unfortunately the invariant coupling of $\boldsymbol{\pi}$ and σ', which we have used for the nucleon and which gives the nucleon mechanical mass through the coupling to σ', cannot be applied to an isotopic singlet or triplet like the Λ and Σ. We may, of course, make use of global symmetry ([18]) or a restricted version of it in which Λ and Σ are degenerate, so that they can be treated as a pair of doublets. But then all our theorems are approximate, violated by the mechanism that splits Λ and Σ; and the idea that the splitting interactions are « medium strong » and not very important has not received much experimental support.

(*) J. BERNSTEIN, M. GELL-MANN and L. MICHEL: *Nuovo Cimento*, **16**, 560 (1960).

([18]) M. GELL-MANN: *Phys. Rev.*, **106**, 1296 (1957) and J. SCHWINGER: ref. ([16]).

6. – The non-linear model.

Let us consider the possibility of modifying the σ model by making the σ field a function of the π field rather than the field of a new particle. We want, however, to preserve the invariance (in eq. (38)) of the strong interaction Lagrangian $\mathscr{L}_2$ (except for the term $-(\mu_0^2/2f_0)\sigma'$) under four-dimensional rotations among π_x, π_y, π_z and σ'. Thus the only condition we can apply to $\boldsymbol{\pi}$ and σ' is the condition:

$$\pi^2 + \sigma'^2 = C^2 , \tag{43}$$

where C is a constant. If we define g_0 to be positive, then we must take the negative square root for σ':

$$\sigma' = -\sqrt{C^2 - \pi^2} ,$$

in order to have a positive mass term for the nucleon. If, when g_0 tends to zero, this mass term is to be simply m_0, then C^2 must be $1/4f_0^2$ so that $g_0\sqrt{C^2} = m_0$. We have, then,

$$\sigma' = -\sqrt{\frac{1}{4f_0^2} - \pi^2} = -\frac{1}{2f_0}\sqrt{1 - 4f_0^2\pi^2} . \tag{44}$$

Instead of the Lagrangian $\mathscr{L}_2$, we have:

$$\mathscr{L}_3 = \left\{ -\bar{N}[\gamma\,\partial - g_0(\sigma' + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)]\,N - \frac{(\partial\boldsymbol{\pi})^2}{2} - \frac{(\partial\sigma')^2}{2} - \frac{\mu_0^2}{2f_0}\sigma' \right\} \tag{45}$$

$$\sigma' = -\frac{1}{2f_0}\sqrt{1 - 4f_0^2\pi^2} ,$$

to within a constant.

This Lagrangian can also be derived by another, slightly more general, method. We can modify the usual pseudoscalar coupling theory by changing every constant into an arbitrary function of π^2:

$$\mathscr{L}_3' = -\bar{N}\gamma\,\partial N - m_0(\pi^2)\bar{N}N + ig_0(\pi^2)\bar{N}\gamma_5\tau\cdot\pi N - \mu_0^2(\pi^2)\frac{\pi^2}{2} - \frac{1}{2}F_{ij}(\pi^2)\,\partial_\mu\pi_i\,\partial_\mu\pi_j ,$$

with

$$F_{ij}(\pi^2) = f_1(\pi^2)\,\delta_{ij} + f_2(\pi^2)\,\pi_i\pi_j .$$

This general expression contains five different *functions* of π^2 instead of only

one as in the previous derivation (*i.e.* $\sigma'(\pi^2)$). We now require that equations of motion have as a consequence eq. (27), and that $\mathscr{L}_3'$ tend to the usual pseudoscalar coupling theory when $g_0 \to 0$. It then follows that the five arbitrary functions are determined uniquely by these requirements and that the corresponding Lagrangian coincides with $\mathscr{L}_3$ to within a constant.

The new Lagrangian represents a theory of the strong interactions of nucleons and pions different from both the gradient coupling and the pseudoscalar coupling theories. We may re-express it as follows:

$$(46) \quad \mathscr{L}_3 = -\bar{N}[\gamma^\mu \partial_\mu + m_0\sqrt{1-4f_0^2\pi^2} - ig_0\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5]N - \frac{(\partial\boldsymbol{\pi})^2}{2} - \frac{2f_0^2|\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}|^2}{1-4f_0^2\pi^2} + \frac{\mu_0^2}{4f_0^2}\left(\sqrt{1-4f_0^2\pi^2}-1\right),$$

again to within a constant. Expanding the Lagrangian to first order in the coupling constant, we have just the pseudoscalar coupling theory, but the remaining orders modify the interaction and destroy its renormalizability in perturbation theory (*).

It is conceivable that the theory may somehow be renormalizable anyway. Suppose we consider the Lagrangian $\mathscr{L}_2$, which is certainly renormalizable, and express all the various amplitudes as functional integrals over classical field variables $\boldsymbol{\pi}$ and σ'. The results of the new theory are obtained from those of the σ theory simply by incorporating in the functional integrals a δ-function of $\sigma' + \sqrt{1/4f_0^2 - \pi^2}$. It is hard to see how this restriction of the integrations can really render the theory *more* singular.

We may think of the restriction $\pi^2 + \sigma'^2 = 1/4f_0^2$ as resulting simply from a choice of the parameter λ_0 in the Lagrangian $\mathscr{L}_2'$ of the σ model. If we take $\lambda_0 = +\infty$, then that corresponds, at least classically, to an infinitely steep potential well for the quantity $\pi^2 + \sigma'^2 - (1/4f_0^2)$, confining it to the value zero.

It should be noted that in the non-linear theory the higher order corrections to the pseudoscalar coupling Lagrangian are perhaps such as to improve agreement with experiment. We know that in the pseudoscalar theory in second order the scattering length for zero-momentum, zero-energy pions on nucleons is $-g^2/m$, while experimentally the low energy S-wave π-N scattering amplitude without charge exchange is very small. We can see, though, that in the Lagrangian $\mathscr{L}_3$ the second order term $2m_0 f_0^2 \bar{N}N\pi^2$ just cancels out the second order effect of the first order coupling. It seems that the cancellation of obnoxious terms like g^2/m, g^4/m, etc., occurs to all orders.

(*) J. BERNSTEIN, M. GELL-MANN and L. MICHEL: *Nuovo Cimento*, **16**, 560 (1960).

There is perhaps some hope, then, that the non-linear Lagrangian might lead to a small S-wave π-$\mathcal{N}$ scattering amplitude without charge exchange in perturbation theory. The lowest order S-wave amplitude with charge exchange is in any case of the right order of magnitude, as in the pseudoscalar and gradient coupling theories.

The other new term that appears in second order is $-2f_0^2(\boldsymbol{\pi}\cdot\partial_\lambda\boldsymbol{\pi}^2) - -(\mu_0^2/2)f_0^2(\pi^2)^2$, which describes π-π scattering with an unrenormalized amplitude of the order of $\mu_0^2 f_0^2$; it is interesting that CHEW and MANDELSTAM have considered π-π scattering with a renormalized amplitude of the order of $m_\pi^2 f_1^2$.

In the non-linear model, the construction of the weak currents by gauge transformations from the strong coupling Lagrangian goes through much as in the σ model. The important features are that the vector current is still divergenceless, the divergence of the pseudovector current is still proportional to the π field, and the gauge group is essentially the same as before.

It should be added that the π field used here is not of the usual type, since $|\boldsymbol{\pi}|<\frac{1}{2}f_0$. We can transform, however, to a more normal pion field $\tilde{\boldsymbol{\pi}}$ by a simple substitution such as $\boldsymbol{\pi}=\tilde{\boldsymbol{\pi}}(1+f_0^2\tilde{\boldsymbol{\pi}}^2)^{-1}$. Of course $\partial_\lambda \boldsymbol{P}_\lambda$ is still proportional to $\boldsymbol{\pi}$, not to $\tilde{\boldsymbol{\pi}}$. See BERNSTEIN, FUBINI, GELL-MANN and THIRRING (loc. cit.).

This third model belongs to a class of theories recently discussed by GÜRSEY [19], who has particularly emphasized the four-dimensional rotations, although he has not considered isotopic rotations that are functions of space and time.

7. – Symmetry operators of the models.

The symmetry properties of the models can best be exhibited in terms of the operators that generate the gauge transformations which, in turn, generate the weak current $\boldsymbol{P}_\alpha+\boldsymbol{V}_\alpha$. Let us consider the truncated version of each theory in which the term in the Lagrangian proportional to a is suppressed so that the weak current is exactly conserved. We may then construct the constant operator $\boldsymbol{R}$, proportional to $\int \mathrm{d}^3x(P_4+V_4)$, which generates the gauge transformations of the theory with infinitesimal gauge function $\boldsymbol{w}$:

$$\left\{\begin{array}{l} \psi_i(x) \to (1-i\boldsymbol{R}\cdot\boldsymbol{w}(x))\,\psi_i(x)(1+i\boldsymbol{R}\cdot\boldsymbol{w}(x)) \\ \qquad \to \psi_i(x) - i\boldsymbol{w}(x)\cdot[\boldsymbol{R},\,\psi_i(x)]\,, \end{array}\right. \tag{47}$$

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[19] F. GÜRSEY: *Nuovo Cimento*, **16**, 230 (1960).

where ψ_i represents any of the field components. The operator $\boldsymbol{R}$ for the weak current is analogous to the electric charge operator Q for the electromagnetic current. We may, of course, separate the parts of $\boldsymbol{R}$ that correspond to the vector and the pseudovector currents. As long as we stick to the conserved vector current theory (and to our choice of scale for the gauge function), then the first part of $\boldsymbol{R}$ is simply twice the isotopic spin $\boldsymbol{I}$. Let us write:

$$\boldsymbol{R} = 2\boldsymbol{I} + 2\boldsymbol{D}\,, \tag{48}$$

where $2\boldsymbol{D}$ generates the pseudovector gauge transformations.

In the first model, it is easy to see from eq. (30), that $\boldsymbol{D}$ is a translation operator, so that we have the commutation rules:

$$[D_i, D_j] = 0\,, \tag{49}$$

as well as the rules

$$\begin{cases} [I_i, I_j] = i e_{ijk} I_k\,, \\ [I_i, D_j] = i e_{ijk} D_k\,, \end{cases} \tag{50}$$

that follow from $\boldsymbol{I}$ being the isotopic spin and $\boldsymbol{D}$ an isotopic vector. Here e_{ijk} is, of course the Kronecker antisymmetric tensor). The total operator $\boldsymbol{R}$ then has the commutation rules:

$$[R_i, R_j] = 4 i e_{ijk}(I_k + 2D_k) \tag{51}$$

that exhibit the asymmetry between V and A characteristic of the gradient coupling model (*).

In the second and third models, the operator $\boldsymbol{D}$ is not a translation operator; in place of (4*a*) we have the commutation rules:

$$[D_i, D_j] = i e_{ijk} I_k\,, \tag{52}$$

which give for $\boldsymbol{R}$ the very simple rules:

$$[R_i, R_j] = 8 i e_{ijk}(I_k + D_k) = 4 i e_{ijk} R_k\,. \tag{53}$$

(*) Of course, what really counts is the nature of the commutation relations for the *complete* operators that generate the sum of the $\Delta S = 0$ and $\Delta S = 1$ currents.

In other words, $\boldsymbol{R}$ is just four times an angular momentum:

$$\boldsymbol{R} = 4\,\boldsymbol{I}_A\,. \tag{54}$$

But in the same way we can show that $2\boldsymbol{I} - 2\boldsymbol{D}$ is four times an angular momentum $\boldsymbol{I}_B$ and furthermore that $\boldsymbol{I}_A$ and $\boldsymbol{I}_B$ commute. Thus the isotopic spin $\boldsymbol{I}$ can be written as the sum of two commuting angular momenta:

$$\boldsymbol{I} = \boldsymbol{I}_A + \boldsymbol{I}_B\,, \tag{55}$$

and the weak current is just proportional to the current of the « spin » $\boldsymbol{I}_A$. We have just demonstrated the well-known property of the group of rotations in a four-dimensional Euclidean space ([20]), that it can be generated by two commuting angular momenta.

In our second and third models, we have assigned quantum numbers as follows:

$$\left\{\begin{array}{ll} N_L & (\tfrac{1}{2},\,0)\,, \\ N_R & (0,\,\tfrac{1}{2})\,, \\ (\boldsymbol{\pi},\,\sigma') & (\tfrac{1}{2},\,\tfrac{1}{2})\,, \end{array}\right. \tag{56}$$

where we use the notation of Gürsey ([19]), in which $N_L \propto (1+\gamma_5)N$ and $N_R \propto (1-\gamma_5)N$ and the quantum numbers are written in the form (I_A, I_B).

8. – Conclusions.

We have found three models of the strong and weak interactions of nucleons and pions in which the divergence of the axial vector weak current is proprotional to the pion field, and we have shown that this property can explain the decay rate of the charged pion.

The gradient coupling model is highly divergent without a cut-off and the weak interaction is introduced in a way that is unsymmetrical between V and A. However, to extend this model to the $\Delta S = 0$ weak interactions of strange particles is very easy. As long as the source of the pion field is the divergence of a pseudovector, we can always find an axial vector current with the right property.

([20]) A. PAIS: *Proc. Nat. Acad. Sci.*, **40**, 835 (1954).

The σ model is renormalizable and even the matrix elements of the weak coupling seem to be finite. Moreover, the four-dimensional invariance (broken only by one term which is responsible for the nucleon mass and for the non vanishing divergence of the axial vector current) gives complete symmetry between V and A. However the model involves the introduction of a new particle. It also presents difficulties when we try to extend it to the strange particles, because the high symmetry of the coupling to π and σ', while easy to arrange for a fermion of isotopic spin $\frac{1}{2}$ like the nucleon, is hard to imitate Λ or Σ or K, unless, of course, we do it approximately, making use of something like global symmetry.

The non linear model does not appear renormalizable although it might be so in some unusual sense. It avoids, however, the introduction of a new particle, while retaining the symmetry properties of the σ model. The difficulty of extension to the strange particles is of course, the same for both models.

Since all the models seem to have some unpleasant features, we should certainly reconsider whether the formula of Goldberger and Treiman can be plausibly derived without such a stringent condition as eq. (5).

To the extent that one tries to retain eq. (5) or something like it, one might pursue further research along several lines: trying to include the strange particles; trying to renormalize the third model; exploring the connection of our gauge transformations with possible intermediate fields for the weak interactions; seeking to describe the $\Delta S = 1$ weak interactions as well as those with $\Delta S = 0$; and looking for parallels between the weak interactions of leptons and those of baryons and mesons.

In closing, let us emphasize that we wish this work to be considered as a highly tentative effort. We have after all, explained only one experimental number, the charged pion lifetime. We do not want to give the impression that the whole theory of strong and weak interactions should be based on this one number, like a pyramid balanced on one point. We do hope, however, that if this type of investigation is pursued further, it may lead to other predictions or to correlations of experimental data.

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One of us (M. GELL-MANN) would like to thank Prof. ABDUS SALAM for his hospitality at Imperial College, London, and for many valuable conversations. He is grateful to Prof. R. P. FEYNMAN and Drs. E. R. NORTON and W. K. R. WATSON for the discussions of the gradient coupling model that initiated this work. He is also indebted to Dr. J. BERNSTEIN for discussions of gauge invariance.

RIASSUNTO (*)

Allo scopo di dedurre in maniera convincente la formula di Goldberger e Treiman per il tasso di decadimento dei pioni carichi, prendiamo in considerazione la possibilità che la divergenza della corrente vettoriale assiale nel decadimento β sia proporzionale al campo del pione. Si presentano tre modelli della interazione pione-nucleone (e della corrente debole) che hanno la proprietà richiesta. Il primo, che si serve dell'accoppiamento di gradiente, ha il vantaggio di poter essere facilmente generalizzato alle particelle strane, ma gli svantaggi di non essere rinormalizzabile e di introdurre le correnti vettoriale e vettoriale assiale in modo asimmetrico. Il secondo modello, che usa un'interazione forte proposta da SCHWINGER ed una corrente debole proposta da POLKINHORNE, è rinormalizzabile e simmetrico fra V ed A, ma comporta la postulazione di una nuova particella ed è difficilmente estensibile alle particelle strane. Il terzo modello è simile al secondo salvo che non è necessario introdurre una nuova particella. (Si perde, tuttavia, la rinormalizzazione nel senso usuale.) Si suggerisce una ulteriore ricerca su queste linee, compresa la considerazione della possibilità che il tasso di decadimento del pione possa ottenersi in modo plausibile con condizioni meno restrittive.

(*) *Traduzione a cura della Redazione.*